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GELFAND AND WALLMAN-TYPE COMPACTIFICATIONS

CHARLES M. BILES

In this paper we compare the Gelfand and Wallman methods of constructing a compactification for a Tychonoff space X from a suitable ring of continuous real-valued functions on X. Every Hausdorff compactification T of X is Gelfand constructable; in particular, T is equivalent, as a compactification of X, to the structure space of all maximal ideals of the ring of all continuously extendable functions from X to T. However, Wallman's method applied to this ring may not yield T. We thus inquire into some relationships that exist between the Wallman and Gelfand compactification of X constructed from a suitable ring of functions on X.

0. Topological preliminaries. All topological spaces in this paper are assumed to be completely regular and Hausdorff. We shall be concerned with methods of constructing compactifications for such spaces.

Let X be a topological space. The space T is an extension of X means there exists a homeomorphism h from X into T such that $h[X]$ is dense in T. The function h is called an embedding. Occasionally the necessary embedding maps will be explicitly mentioned, but usually they will be tacitly assumed. In fact, when T is given as an extension of X, we may take X as a subspace of T. The space T is a compactification of X (denoted $T \in cX$) means that T is a compact extension of X. The compactifications T and K of a space X are equivalent as compactifications of X (denoted $T = K$) means there exists a homeomorphism between T and K such that $h(x) = x$ for each $x \in X$.

We shall use the standard notations [4] regarding $C(X)$, the ring of continuous real-valued functions. For any $f \in C(X)$,

$$Z(f) = \{x \in X \mid f(x) = 0\}$$

is called the zero-set of f. If $\mathscr{A}$ is a subring of $C(X)$, we define $Z[\mathscr{A}] = \{Z(f) \mid f \in \mathscr{A}\}$; however, $Z[C(X)]$ is customarily denoted by $Z(X)$. We shall only refer to subrings of $C(X)$ with unity.

Let $\mathscr{A}$ be a subring of $C(X)$. We shall denote the space of maximal ideals of $\mathscr{A}$ with the Stone topology [4, 7M], also called the structure space of $\mathscr{A}$, by $H[\mathscr{A}]$. The space of ultrafilters of $Z[\mathscr{A}]$ is denoted by $wZ[\mathscr{A}]$. This space of ultrafilters is constructed by Wallman's method [1] [2]. We shall be primarily concerned with those subrings $\mathscr{A}$ of $C(X)$ for which $wZ[\mathscr{A}] \in cX$ and how these