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SOME REMARKS ON SPECIAL DISCONJUGACY CRITERIA FOR DIFFERENTIAL SYSTEMS

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The present note is devoted to the discussion of a special class of transformations that preserves the oscillatory or disconjugacy nature of solutions for a type of first order differential system in a $2n$-dimensional vector function, and which includes as a particular instance a well-known transformation of a real scalar second order homogeneous ordinary differential equation to the canonical form $y''(t) + \gamma(t)y(t) = 0$.

It is well-known, (see, for example, Reid [4, 6, 7]), that for self-adjoint differential systems conditions of oscillation may be characterized by variational criteria, and, in particular, the property of disconjugacy is equivalent to the positive definiteness of an associated hermitian integral functional on a suitable class of vector functions. Moreover, (see, for example, Reid [4; § 5]; Hartman and Wintner [2]), results for self-adjoint differential systems may be applied to yield sufficient conditions for disconjugacy in the case of nonself-adjoint systems. These criteria and procedures are of basic significance for the utilization of the considered transformations in the study of oscillation and comparison phenomena.

Matrix notation is used throughout; in particular, matrices of one column are called vectors, and for a vector $u = (u_\alpha)$, $(\alpha = 1, \cdots, n)$, the norm $|u|$ is given by $(|u_1|^2 + \cdots + |u_n|^2)^{1/2}$; the linear vector space of ordered n-tuples of complex numbers, with complex scalars, is denoted by $\mathbf{C}_n$. The $n \times n$ identity matrix is denoted by E_n, or by merely E when there is no ambiguity, while 0 is used indiscriminately for the zero matrix of any dimensions; the conjugate transpose of a matrix M is denoted by M^*. If M is an $n \times n$ matrix the symbol $\nu[M]$ is used for the maximum of $|My|$ on the unit ball $\{y: |y| \leqq 1\}$ in $\mathbf{C}_n$. The notation $M \geqq N$, $\{M > N\}$, is used to signify that M and N are hermitian matrices of the same dimensions, and $M - N$ is a nonnegative, {positive} definite hermitian matrix. In general, if M is an $n \times n$ matrix, let $\mathfrak{Re}\, M$ and $\mathfrak{Im}\, M$ denote the hermitian matrices $\mathfrak{Re}\, M = \frac{1}{2}(M + M^*)$, $\mathfrak{Im}\, M = i/2(M^* - M)$ so that $M = \mathfrak{Re}\, M + i\, \mathfrak{Im}\, M$. If the elements of a matrix function $M(t)$ are a.c. (absolutely continuous) on arbitrary compact subintervals of a given interval I, then $M(t)$ is said to be locally a.c. on I; moreover, $M'(t)$ signifies the matrix of derivatives at values where these derivatives exist and the zero matrix elsewhere. Correspondingly, if the elements of $M(t)$ are