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UNBOUNDED INVERSES OF HYPONORMAL OPERATORS

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It is shown that certain unbounded inverses of hyponormal operators have Cartesian representations in which the real part is absolutely continuous and the imaginary part is bounded. An example is given which shows that in general the imaginary part is not absolutely continuous.

A bounded operator T on a Hilbert space $\mathfrak{H}$ is said to be hyponormal if

$$T^*T - TT^* \geqq 0 . \tag{1.1}$$

For properties of such operators, see Putnam [2]. Such an operator is said to be completely hyponormal if there exists no nontrivial subspace of $\mathfrak{H}$ which reduces T and on which T is normal. Recall that a self-adjoint operator A with the spectral resolution $A = \int \lambda dE_\lambda$ is said to be absolutely continuous if $\|E_\lambda x\|^2$ is an absolutely continuous function of λ for all x in $\mathfrak{H}$. If T is completely hyponormal with the Cartesian representation

$$T = H + iJ , \tag{1.2}$$

then both H and J are absolutely continuous; see [2, p. 42].

In case 0 is not in the spectrum of T then T^{-1} is also hyponormal; Stampfli [7]. Further,

$$\|T^{-1}\| = d^{-1} \text{ and } \|Tx\| \geqq d\|x\|, x \in \mathfrak{H} \text{ and } d = \operatorname{dist}(0, \operatorname{sp}(T)) . \tag{1.3}$$

Suppose however that 0 is in the continuous spectrum of T, so that T^{-1} exists as an unbounded operator, is closed, and $\mathfrak{D}_{T^{-1}} = \mathfrak{R}_T$ is dense in $\mathfrak{H}$; cf. Stone [9, pp. 40, 129]. Then it was shown by Stampfli [8] that

$$\mathfrak{D}_{T^{-1}} \subset \mathfrak{D}_{T^{-1*}} \text{ and } \|T^{-1*}x\| \leqq \|T^{-1}x\| \text{ for } x \in \mathfrak{D}_{T^{-1}} . \tag{1.4}$$

Thus T^{-1} still behaves to a certain extent as does T. The question arises however as to whether T^{-1} admits a Cartesian representation $T^{-1} = K + iL$, where K and L are self-adjoint, and also, if such a representation exists, whether these operators are absolutely continuous when T is completely hyponormal. A partial answer is contained in the theorem below.