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ON A THEOREM OF M. IZUMI AND S. IZUMI

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This paper establishes a theorem on the absolute Nörlund summability of Fourier series which generalizes and unifies generalizations by the author and by M. and S. Izumi of an earlier result by McFadden.

Let $\sum a_n$ be a series with partial sums S_n and let p_n be a sequence of real constants with

$$P_n = \sum_{v=0}^{n} p_v\ , \quad p_0 > 0\ , \quad P_{-1} = p_{-1} = 0\ .$$

The series $\sum a_n$ is said to be summable $|N, p_n|$ if

$$\sum_{n=1}^{\infty} |t_n - t_{n-1}| < \infty\ ,$$

where

$$t_n = \frac{1}{P_n} \sum_{v=0}^{n} p_{n-v} S_v\ . \tag{1.1}$$

We write $P(t) = P_{[t]}$ and in the sequel we assume that p_n is nonnegative, nonincreasing and $\lim_{n\to\infty} p_n = 0$.

2. Let $f(t)$ be a periodic function with period 2π and integrable (L) in $(-\pi, \pi)$. The Fourier series of $f(t)$ is

$$\frac{1}{2}a_0 + \sum_{n=1}^{\infty} (a_n \cos nt + b_n \sin nt) \equiv \sum_{n=0}^{\infty} A_n(t)\ ,$$

where a_n and b_n are given by the usual Euler-Fourier formulae. We write

$$\phi(t) = f(x+t) + f(x-t) - 2f(x)\ ,$$

$$\alpha(t) = \sum_{v=0}^{\infty} p_v \cos vt\ , \quad \beta(t) = \sum_{v=0}^{\infty} p_v \sin vt\ ,$$

$$\alpha_n = \int_0^{\pi} \phi(t)\alpha(t) \cos nt\, dt\ , \quad \beta_n = \int_0^{\pi} \phi(t)\beta(t) \sin nt\, dt\ ,$$

$$w(\delta) = \sup_{0 \leq |t| \leq \delta} |f(x+t) - f(x)|\ .$$

p and q are mutually conjugate indices in the sense that $1/p + 1/q = 1$.

Recently M. Izumi and S. Izumi ([2, Th. 3]) proved the following