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SOME TOPOLOGICAL PROPERTIES WEAKER THAN COMPACTNESS

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Many topological properties may be described by covering relations which may also generally be easily described in terms of filter relations. For example, a space is compact if and only if each open cover of the space contains a finite subcover, or equivalently, if each filter has an adherent point. In this paper, characterizations are given of some topological properties weaker than compactness, both in terms of filters and coverings. In the final section a question posed by Viglino and by Dickman and Zame is answered.

2. **Definitions and notations.** (a) A space for which distinct points may be separated by disjoint closed neighborhoods (i.e., a Urysohn space) will be labeled a $T_{2(1/2)}$-*space*. Let $\nu = 2, 2\frac{1}{2}$, or 3. A T_ν-space is said to be T_ν-*closed* if it is closed in each T_ν-extension. A T_ν-space (X, τ) is said to be T_ν-*minimal* if there exists no T_ν topology on X strictly weaker than τ.

(b) A Hausdorff space (X, τ) is *C-compact* if given a closed set Q of X and a τ-open cover $\mathscr{O}$ of Q, then there exists a finite number of elements of $\mathscr{O}$, say 0_i, $1 \leqq i \leqq n$, with $Q \subset \mathrm{cl}_X \bigcup_{i=1}^{n} 0_i$.

(c) A Hausdorff space X is *functionally compact* if for every open filter $\mathscr{U}$ in X such that the intersection A of the elements of $\mathscr{U}$ equals the intersection of the closures of the elements of $\mathscr{U}$, then $\mathscr{U}$ is the neighborhood filter of A.

(d) A filter is *open* (closed) if it has a base of open (closed) sets. A *regular filter* is a filter which is both open and closed.

(e) Let A be a subset of a space X. An open cover, $\mathscr{U}$, of A will be said to be a *Urysohn cover* if for each $x \in A$ there exist elements $0_1, 0_2$ of $\mathscr{U}$ with $x \in 0_1 \subset \mathrm{cl}\, 0_1 \subset 0_2$.

(f) Let A be a subset of a space X. An open cover, $\mathscr{S}$, of A will be said to be a *strong cover* if for each $x \in A$ there exist $\{0_n\}_{n=1}^{\infty} \subset \mathscr{S}$ with $x \in 0_1$ and $\mathrm{cl}\, 0_i \subset 0_{i+1}$ for each i.

(g) A closed subset Y of a space X is *regular closed* if given $x \in X \backslash Y$, then there exists an open set 0 with $x \in 0 \subset \mathrm{cl}\, 0 \subset Y^c$.

3. **Covering theorems.** Filter characterizations for T_ν-closed and T_ν-minimal spaces are listed below. The proof of (a) may be found in [2]; (b) in [1]; (c), (d), and (e) in [5]; (f) in [4].

THEOREM I. (a) *A T_2-space is T_2-closed if and only if every*