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ON THE IDEAL STRUCTURE OF SOME ALGEBRAS OF ANALYTIC FUNCTIONS

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Using the Beurling-Lax description of invariant subspaces of $H^2(R)$, we describe the ideal structure of two large classes of convolution algebras whose Fourier-Laplace Transforms are entire functions. A closed ideal will be characterized by its cospectrum or by its cospectrum together with a nonnegative number related to the "rate of decrease at infinity"; in the latter case, the closed ideals having the same cospectrum form a totally ordered family $\{I_\xi\}$, $\xi \in [0, \infty)$, with $I_\xi \supsetneqq I_\eta$ whenever $\xi < \eta$. New examples of algebras to which the results apply are given.

The familiar notation for the spaces considered by Schwartz ([9]) is adopted and each space is equipped with its usual topology. Let $\mathscr{K}$ be the subspace of $\mathscr{E}(R)$ of functions ϕ for which

$$||\phi||_k = \sup_{x \in R, p \leqq k} \exp(k|x|)|D^p\phi(x)|$$

is finite for each $k = 0, 1, \cdots$; the topology on $\mathscr{K}$ will be the one induced by the semi-norms $||(\cdot)||_k$, $k = 0, 1, \cdots$. Under this topology $\mathscr{K}$ is a convolution algebra with separately continuous multiplication. A detailed discussion of $\mathscr{K}$ along with associated spaces is given in [4], [12] and [13] (note that Zielézny uses $\mathscr{K}_1$ instead of $\mathscr{K}$). We recall some of the results in the form most convenient for applications here.

Denote by $\mathscr{O}_c'(\mathscr{K})$ the convolution operators on $\mathscr{K}$, i.e., the distributions $S \in \mathscr{D}'(R)$ for which the convolution operator $\phi \rightarrow S * \phi$ is well-defined and continuous from $\mathscr{K}$ into $\mathscr{K}$. $\mathscr{O}_c'(\mathscr{K})$ is given the topology it inherits as a subspace of $\mathscr{L}_b(\mathscr{K}, \mathscr{K})$, the continuous linear mappings from $\mathscr{K}$ into $\mathscr{K}$, when $\mathscr{L}_b(\mathscr{K}, \mathscr{K})$, has the topology of uniform convergence on bounded subsets of $\mathscr{K}$. Alternatively, if $\mathscr{K}'$ is the strong dual of $\mathscr{K}$, $\mathscr{O}_c'(\mathscr{K})$ can be defined as the space $\mathscr{O}_c'(\mathscr{K}', \mathscr{K}')$ of convolution operators on $\mathscr{K}'$ in the sense of Schwartz ([10], exposé 10) and given the topology acquired as a subspace of $\mathscr{L}_b(\mathscr{K}', \mathscr{K}')$. These two definitions of $\mathscr{O}_c'(\mathscr{K})$ are, however, entirely equivalent (cf. [13, Ths. 2(d'), 4]).

THEOREM 1. *The space $\mathscr{O}_c'(\mathscr{K})$ is a convolution algebra for which*
(i) $(S, T) \rightarrow S * T$ *is a separately continuous mapping from* $\mathscr{O}_c'(\mathscr{K}) \times \mathscr{O}_c'(\mathscr{K})$ *into* $\mathscr{O}_c'(\mathscr{K})$,