PACIFIC JOURNAL OF MATHEMATICS
Vol. 36, No. 1, 1971

A NONSTANDARD PROOF OF THE JORDAN CURVE THEOREM

LOUIS NARENS

In this paper a proof of the Jordan curve theorem will be presented. Some familiarity with the basic notions of nonstandard analysis is assumed. The rest of the paper is self-contained except for some standard theorems about polygons.

The theorem will be proved in what ought to be a natural way: by approximation by polygons. This method is not usually found in the standard proofs since the approximating sequence of polygons is often unwieldly. But by using nonstandard analysis, one can approximate a Jordan curve by a single polygon that is infinitesimally close to the curve. This allows types of reasoning which are extremely difficult and unnatural on sequences of polygons.

Preliminaries. The basic concepts of nonstandard analysis and some acquaintance with polygons are assumed. Some basic definitions and theorems of point set topology are also assumed.

Throughout this paper the following notations and conventions will be used:

(1) All discussion, unless otherwise stated, is assumed to be about a nonstandard model of the Euclidean plane. "Otherwise stated" will often mean that the notion or concept will be prefaced by the word "standard".

(2) A standard concept and its extension will be denoted by the same symbol. If it is necessary to distinguish between them, reference to the model in which they are to be interpreted will be made.

(3) If A and B are sets of points and x is a point, then $|x, A|$ will denote the distance from x to A and $|B, A| = \inf_{x \in B} |x, A|$. (Thus if $A \cap B \neq \emptyset$ then $|A, B| = 0$.) $|x, y|$ will denote the distance from the point x to the point y.

(4) f will denote a fixed continuous function on $[0, 1]$ into the Euclidean plane with the property that $x < y$ and $f(x) = f(y)$ if and only if $x = 0$ and $y = 1$. C will denote the range of f.

(5) $x \cong y$ will mean that the distance from x to y is infinitesimal. If x is near-standard then ${}^\circ x$ will denote the standard y such that $x \cong y$.

(6) If x and y are points then xy will denote the *ordered*, closed line segment that begins at x and ends at y.

(7) If x and y are points then $\operatorname{intv}(x, y)$ is the set of all points z of xy such that $z \neq x$ and $z \neq y$.