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LOCAL RINGS WITH NOETHERIAN FILTRATIONS

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The purpose of this paper is to investigate a certain type of complete filtered local rings, and to generalize well-known theorems in literature which are related to the structure theorem of complete local rings.

All rings in this article are commutative with identity element, and all subrings possess the identity element of the containing ring. A ring A is a local ring if it has a unique maximal ideal m, symbolically we write (A, m), and by the natural topology of (A, m) we mean the m-adic topology.

A filtration $\{q_n\}_{n=0}^{\infty}$ of a ring R is called a noetherian filtration if it is separated and R/q_n is a noetherian ring for all n. Motivated by I. S. Cohen's structure theorem of complete noetherian local rings with respect to the natural topologies ([5]), extension of two theorems on noetherian local rings is considered. Harrison-Kolman's theorem [6, Th. A, Th. 1], which is annalogous to Cohen's theorem, is generalized as follows: If R is a local ring which is complete with respect to a noetherian filtration $\{q_n\}_{n=0}^{\infty}$ such that (i) R is equicharacteristic and (ii) R/q_1 is regular, then there exists a subring B such that $B + q_1 = R$ and $B \cap q_1 = (0)$; consequently, R is a homomorphic image of a ring of formal power series in a finite or countably infinite number of indeterminates over B. We also prove that if R is a complete (resp. compact) local ring with respect to a noetherian filtration $\{q_n\}_{n=0}^{\infty}$ such that q_1 is its maximal ideal, then R is a homomorphic image of a ring $k[[x_i]]_{i\in I}$ of formal power series in a finite or countably infinite number of indeterminates over a complete (resp. compact) noetherian local subring k for the natural topology; conversely, such power series ring possesses a noetherian filtration and $k[[x_i]]_{i\in I}/D$, for any closed ideal D of $k[[x_i]]_{i\in I}$ relative to the filtration topology, is a complete (resp. compact) local ring with respect to a noetherian filtration. This is an extension of Cohen-Warner's theorem ([5, Th. 9, 12]; [11, Th. 14]). Finally, we apply the result to prove that a certain class of local rings are unique factorization domains.

1. **Natural topologies of power series rings.** Let N be the set of all positive integers, and $N_0 = N \cup \{0\}$. Throughout the paper, we shall denote the power series ring $R[[x_1, x_2, \cdots, x_t]]$ in t indeterminates over a ring R by R_t for each $t \in N_0$, identifying R_0 with R. Also the power series ring $R[[x_1, x_2, \cdots]] = R[[x_i]]_{i\in N}$ in a countably