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ON BERGMAN OPERATORS FOR PARTIAL DIFFERENTIAL EQUATIONS IN TWO VARIABLES

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Bergman operators are linear integral operators that map complex analytic functions into solutions of linear partial differential equations with analytic coefficients. In this way methods and results of complex analysis can be used for characterizing general properties of classes of those solutions. For example, this approach yields theorems about the location and type of singularities, the growth, and the coefficient problem for series developments of solutions.

A partial differential equation being given, there exist various types of Bergman operators, and for that purpose it is essential to select an operator whose generating function is as simple as possible. The present paper considers differential equations in two independent variables, introduces a class of Bergman operators satisfying that requirement, and determines the corresponding class of differential equations in an explicit fashion. In fact, necessary and sufficient conditions are obtained in order that the solutions of a partial differential equation can be obtained by means of a Bergman operator of that class. It is also shown that the set of these equations includes several equations of practical importance.

2. Bergman operators of class P_0. We consider partial differential equations of the form

$$\Delta\psi + \alpha(x, y)\psi_x + \beta(x, y)\psi_y + \gamma(x, y)\psi = 0$$

assuming that α, β and γ are real analytic functions in some neighborhood of the origin. Setting $z_1 = x + iX$, $z_2 = y + iY$, we may continue the coefficients to complex values of the variables. We now introduce the variables

$$z = z_1 + iz_2 \quad \text{and} \quad z^* = z_1 - iz_2\,.$$

(Note that $z^* = \bar{z}$ if z_1 and z_2 are real.) Transforming the given equation and eliminating one of the two first partial derivatives, we obtain

$$Lu := u_{zz^*} + b(z, z^*)u_{z^*} + c(z, z^*)u = 0\,. \tag{2.1}$$

A Bergman operator B corresponding to (2.1) may be defined by means of