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EMBEDDINGS IN MATRIX RINGS

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For a fixed integer $n \geqq 1$, and a given ring R there exists a homomorphism $\rho; R \to M_n(K)$, K a commutative ring such that every homomorphism of R into an $n \times n$ matrix ring $M_n(H)$ over a commutative ring can be factored through ρ by a homomorphism induced by a mapping $\eta: K \to H$. The ring K is uniquely determined up to isomorphisms. Further properties of K are given.

1. Notations. Let R be an (associative) ring, $M_n(R)$ will denote the ring of all $n \times n$ matrices over R. If $\eta: R \to S$ is a ring homomorphism then $M_n(\eta)$: $M_n(R) \to M_n(S)$ denotes the homomorphism induced by η on the matrix ring, i.e., $M_n(\eta)(r_{ik}) = (\eta(r_{ik}))$.

If $A \in M_n(R)$, we shall denote by $(A)_{ik}$ the entry in the matrix A standing in the (i, k) place.

Let k be a commutative ring with a unit (e.g., $k = \mathbf{Z}$ the ring of integers). All rings considered henceforth will be assumed to be k-algebras on which $1 \in k$ acts as a unit, and all homomorphisms will be k-homomorphisms, and will be into unless stated otherwise.

Let $\{x_i\}$ be a set (of high enough cardinality) of noncommutative indeterminates over k, and put $k[x] = k[\cdots, x_i, \cdots]$ the free ring generated over k with k commuting with the x_i. We shall denote by $k^0[x]$ the subring of $k[x]$ containing all polynomials with free coefficient zero.

Denote by $X_i = (\xi^i_{\alpha,\beta})$ $\alpha, \beta = 1, 2, \cdots, n$ the generic matrices of order n over k, i.e., the elements $\{\xi^i_{\alpha,\beta}\}$ are commutative indeterminates over k. Let $\Delta = k[\xi] = k[\cdots, \xi^i_{\alpha,\beta}, \cdots]$ denote the ring of all commutative polynomials in the ξ's, then we have $k^0[X] \subseteqq k[X] \subseteqq M_n(\Delta)$ where $k[X]$ is the k-algebra generated by 1 and all the X_i; $k^0[X]$ is the k-algebra generated by the X_i (without the unit).

There is a canonical homomorphism ψ_0: $k[x] \to k[X]$ which maps also $k^0[x]$ onto $k^0[X]$ given by $\psi_0(x_i) = X_i$.

2. Main result. The object of this note is to prove the following:

THEOREM 1. *Let R be a k-algebra, then*

(i) *There exists a commutative k-algebra S and a homomorphism ρ: $R \to M_n(S)$ such that:*

(a) *The entries $\{[\rho(r)]_{\alpha\beta}; r \in R\}$ generate together with 1, the ring S.*