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# ON THE CONJUGATING REPRESENTATION OF A FINITE GROUP

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**A natural permutation representation for any finite group is the conjugating representation $T$: for each $g \in G$, $T(g)$ is the permutation on the set $\{x \mid x \in G\}$ given by $T(g)(x) = gxg^{-1}$. Frame, Solomon and Gamba have studied some of its properties. This paper considers the question of which complex irreducible representations occur as components of $T$, in particular the conjecture that any such representation whose kernel contains the center of $G$ is a component of $T$. This conjecture is verified for a few special cases and a number of related results are obtained, especially with respect to the one-dimensional components of $T$.**

In §2 we see that the conjecture does hold for groups of "central type" which were studied by DeMeyer and Janusz in [4]. In §3 we obtain further information with respect to the linear characters of $G$; it is shown that if $G/H$ is a cyclic group then the number of irreducible characters of $G$ which are induced from irreducible characters of $H$ is the same as the number of conjugacy classes of $G$ having the property that the centralizers of their elements belong to $H$. This number is precisely the multiplicity in the conjugating representation of a linear character of $G$ whose kernel is $H$.

NOTATION. $G$ is a finite group with conjugacy classes $C_1, C_2, \cdots, C_k$. $\chi^1, \chi^2, \cdots, \chi^k$ are the irreducible complex characters of $G$. $\{g_1, g_2 \cdots, g_k\}$ will be a set of representatives of the conjugacy classes with $g_j \in C_j$ for $j = 1, 2, \cdots, k$. We let $T$ denote the conjugating representation of $G$ defined above and $\theta$ will be the character of $G$ corresponding to $T$. The transitivity classes (orbits) under $T$ are then $C_1, \cdots, C_k$ and restricting $T$ to the set $C_i$ gives the corresponding transitive permutation representation $T^i$ where $i = 1, 2, \cdots, k$. Let $\varphi^i$ be the character of $T^i$ for each $i$, so that $\theta = \sum_{i=1}^{k} \varphi^i$.

If $\eta$ and $\lambda$ are two complex-valued characters on $G$, then $(\eta, \lambda)$ will denote the usual "inner product" given by

$$(\eta, \lambda) = |G|^{-1} \sum_{g \in G} \eta(g) \overline{\lambda(g)}$$

where $\overline{\lambda(g)}$ is the complex conjugate of $\lambda(g)$, and $|G|$ is the order of $G$. $Z$ will denote the center of the group $G$. The kernel of $\lambda$, denoted $\operatorname{Ker} \lambda$, is to mean the kernel of a representation affording the