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A DECOMPOSITION THEOREM FOR TOPOLOGICAL GROUP EXTENSIONS

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G. W. Mackey has developed a characterization of the group of equivalence classes of extensions of a fixed group G by a fixed abelian group A restricting all groups to be locally compact second countable Hausdorff spaces. Calvin C. Moore incorporated his results into a cohomology theory of group extensions such that the second cohomology group, $H^2(G, A)$, coincides with Mackey's group of extension classes. The purpose of this paper is to consider the special case in which G and A are connected, A is a Lie group, G is locally arcwise connected and locally simply connected. Under these conditions G and A admit universal covering groups U_G and U_A. Allowing $\pi(G)$ and $\pi(A)$ to denote the fundamental groups of G and A respectively (with basepoint the identity) any extension of G by A determines an extension of U_G by U_A and an extension of $\pi(G)$ by $\pi(A)$ uniquely up to equivalence. Hence there is a map Φ, in fact a homomorphism, constructed from $H^2(G, A)$ to $H^2(U_G, U_A) \oplus H^2(\pi(G), \pi(A))$. In this paper $H^2(G, A)$ is determined as a direct sum of subgroups of $H^2(U_G, U_A)$ and $H^2(\pi(G), \pi(A))$, and of a third group which is computed.

Throughout all topological groups are assumed to be locally compact, Hausdorff and second countable.

Let G and A be topological groups, A abelian. A is called a G-module if there is given an action of G on A, that is, a continuous function from $G \times A$ into A, carrying (g, a) onto ga, such that (i) for any fixed $g \in G$, the mapping of A into A determined by $a \longrightarrow ga$ is an automorphism, (ii) the automorphism thus determined by the identity of G is the identity automorphism, and (iii) for any g and h in G and $a \in A$, $(gh)a = g(ha)$. If A is a G-module then an extension of G by A is an exact sequence

$$D: 1 \longrightarrow A \xrightarrow{\ i\ } E \xrightarrow{\ j\ } G \longrightarrow 1$$

such that E is a topological group, $i: A \longrightarrow i(A)$ is a homeomorphism and $E/i(A)$ is topologically isomorphic to G or equivalently, j is open, and finally, the action of G on A defined by $xa = i^{-1}(xi(a)x^{-1})$coincides with the given action of G on A. Sometimes, when G and A are fixed, and when no confusion can arise, the extension D will be denoted by $D(E, i, j)$. Two extensions $D(E, i, j)$ and $D'(E', i', j')$, of G by A are called equivalent if there is a topological isomorphism ϕ of