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INTERPOLATION IN C(Ω)

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It is known from the work of Bade and Curtis that if $\mathfrak{A}$ is a Banach subalgebra of $C(\Omega)$, Ω a compact Hausdorff space, and if Ω is an F-space in the sense of Gillman and Hendriksen then $\mathfrak{A} = C(\Omega)$. This paper is concerned with the extension of this and similar results to the setting of Grothendieck spaces (G-spaces for short). An important feature of the extension is that emphasis is shifted from the underlying topological structure of Ω to the linear topological character of $C(\Omega)$.

As a corollary we show that if Ω_1 and Ω_2 are infinite compact Hausdorff spaces, then $\Omega_1 \times \Omega_2$ is not a G-space. Consequently if Ω is a G-space then $C(\Omega)$ is not linearly isomorphic to $C(\Omega \times \Omega)$.

If A is a commutative Banach algebra whose spectrum is a totally disconnected G-space, a second corollary of our extension is that the Gelfand homomorphism is onto. This establishes for G-spaces a result due to Seever for N-spaces.

Two definitions of G-space are to be found in the literature.

(A) A Banach space X is a G-space if every weak-* convergent sequence in X^*, the dual of X, is weakly convergent.

(B) A compact Hausdorff space Ω is a G-space if $C(\Omega)$ is a G-space in the sense of (A).

Unless otherwise noted we shall accept (B) as our definition.

It is known from the work of Seever [7] that if Ω is an F-space, i.e., if disjoint open F_σ subsets of Ω have disjoint closures, then Ω is a G-space. A result due to Rudin [3] states that if Ω_1 and Ω_2 are infinite compact Hausdorff spaces then $\Omega_1 \times \Omega_2$ is not an F-space. Corollary 2.6 is an extension of this to G-spaces. Although an example of a G-space which is not an F-space is given in [7], no necessary and sufficient topological characterization of the G property is known.

1. Preliminaries. Let $M(\Omega)$ be the space of regular Borel measures on Ω equipped with the total variation norm. A sequence $\{\mu_n\}$ in $M(\Omega)$ converges for the weak-* topology if for each f in $C(\Omega)$, the space of continuous complex valued functions on Ω, the sequence $\{\mu_n(f)\}$ is convergent. Weak convergence of $\{\mu_n\}$ means convergence of $\{\gamma(\mu_n)\}$ for every γ in $M^*(\Omega)$, the dual of $M(\Omega)$. If Ω is any set $l_1(\Omega)$ will denote the Banach space of point mass measures on Ω with the total variation norm.

A Banach subalgebra (subspace) $\mathfrak{A}$ of $C(\Omega)$ is a subalgebra (subspace)