PACIFIC JOURNAL OF MATHEMATICS
Vol. 36, No. 3, 1971

A NON-COMPACT KREIN-MILMAN THEOREM

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This paper describes a class of closed bounded convex sets which are the closed convex hulls of their extreme points. It includes all compact ones and those with the positive binary intersection property.

Let K be a closed bounded convex subset of a Hausdorff locally convex linear topological space F. Denote by EK the extreme points of K, by $\mathrm{co}\, EK$ their convex hull and let $\overline{\mathrm{co}}\, EK$ be its closure. We are interested in showing when

$$K = \overline{\mathrm{co}}\, EK .$$

The principal known results are the following:

THEOREM 1.1. *If either*
(a) *K is compact;*
or (b) *K has the positive binary intersection property;*
then
$$K = \overline{\mathrm{co}}\, EK .$$

Case (a) is the Krein-Milman Theorem [3, p. 131]. Case (b) was proved by Nachbin in [6], and he poses in [5, p. 346] the problem of obtaining a theorem of which both (a) and (b) are specializations. This is answered by Theorem 4.2. For the whole of this paper, S is a Stonean (extremally disconnected compact Hausdorff) space.[1]

A simplified version of Theorem 4.2 reads as follows:

THEOREM 1.2. *Let X be a normed linear space. Then any norm-closed ball in the space $\mathfrak{B}(X, C(S))$ of continuous linear operators from X to $C(S)$ is the closure of the convex hull of its extreme points in the strong neighborhood topology.*

The result concerning the unit ball of a dual Banach space in its weak*-topology and that concerning the unit ball in $C(S)$ in its norm topology are special cases of Theorem 1.2.

A *sublinear function* P from a vector space X to a partially ordered space V satisfies

$$P(x + y) \leqq P(x) + P(y)$$

and

[1] Theorem 2.3 and its proof are valid when S is zero-dimensional.