PACIFIC JOURNAL OF MATHEMATICS
Vol. 36, No. 3, 1971

DIFFERENTIAL SIMPLICITY AND COMPLETE INTEGRAL CLOSURE

YVES LEQUAIN

Let R be an integral domain containing the rational numbers, and let R' denote the complete integral closure of R. It is shown that if R is differentiably simple, then R need not be equal to R', even when R is Noetherian, and then the relationship between R and R' is studied.

Let $\mathscr{D}$ be any set of derivations of R. Seidenberg has shown that the conductor $C = \{x \in R \mid xR' \subset R\}$ is a $\mathscr{D}$-ideal of R, so that when R is $\mathscr{D}$-simple and $C \neq 0$, then $R = R'$. We investigate here the situation when $C = 0$.

The first observation that one must make is that it is no longer true that $R = R'$ when R is differentiably simple, even when R is Noetherian. We show this in Example 2.2 where we construct a 1-dimensional local domain containing the rational numbers which is differentiably simple but not integrally closed. This counterexamples a conjecture of Posner [4, p. 1421] and also answers affirmatively a question of Vasconcelos [6, p. 230].

Thus, it is not a redundant task to study the relationship between a differentiably simple ring R and its complete integral closure. An important tool in this study is the technique of § 3 which associates to any prime ideal P of R containing no D-ideal a rank-1, discrete valuation ring centered on P; by means of this, we show in Theorem 3.2 that over such a prime ideal P of R there lies a unique prime ideal of R'. When R is a Noetherian $\mathscr{D}$-simple ring with $\{P_\alpha\}_{\alpha \in \Lambda}$ as set of minimal prime ideals, Theorem 3.3 asserts that $R' = \bigcap_{\alpha \in \Lambda} \{R_\alpha \mid R_\alpha$ is the valuation ring associated with the minimal prime ideal $P_\alpha\}$; Corollary 3.5 asserts that R' is the largest $\mathscr{D}$-simple overring of R having a prime ideal lying over every minimal prime ideal of R.

1. Preliminaries. Our notation and terminology adhere to that of Zariski-Samuel [7] and [8]. Throughout the paper we use R to denote a commutative ring with 1, K to denote the total quotient ring of R, and A to denote an ideal of R; A is proper if $A \neq R$. A derivation D of R is a map of R into R such that

$$D(a + b) = D(a) + D(b) \quad \text{and} \quad D(ab) = aD(b) + bD(a)$$

for all $a, b \in R$.

Such a derivation can be uniquely extended to K, and we shall