PACIFIC JOURNAL OF MATHEMATICS
Vol. 36, No. 3, 1971

RINGS OF QUOTIENTS OF Φ-ALGEBRAS

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Let $\mathscr{X}$ be a completely regular (Hausdorff) space. Fine, Gillman, and Lambek have studied the (generalized) rings of quotients of $C(\mathscr{X}) = C(\mathscr{X}; \mathbf{R})$, with particular emphasis on the maximal ring of quotients, $Q(\mathscr{X})$. In this note, we start with a characterization of $Q(\mathscr{X})$ that differs only slightly from one of theirs. This characterization is easily altered to fit more general circumstances, and so serves to obtain some results on non-maximal rings of quotients of $C(\mathscr{X})$, and to generalize these results to the class of Φ-algebras.

We consider only commutative rings with unit. Let A be one such, and recall that the (unitary) over-ring B of A is called a *rational extension* or *ring of quotients* of A if it satisfies the following condition: given $b \in B$, for every $0 \neq b' \in B$ there is $a \in A$ with $ba \in A$ and $b'a \neq 0$. A ring without proper rational extensions is said to be *rationally complete*. For the rings to be considered here (all are semi-prime), the condition above can be replaced by the simpler condition: for $0 \neq b \in B$, there exists $a \in A$ such that $0 \neq ba \in A$ ([1], p. 5). Accordingly, we make the following

DEFINITION. If B is an over-ring of A and $0 \neq b \in B$, say that b is *rational over* A if there is $a \in A$ with $0 \neq ba \in A$.

Let $m\beta\mathscr{X}$ denote the minimal projective extension of $\beta\mathscr{X}$ and $\tau\colon m\beta\mathscr{X} \to \beta\mathscr{X}$ the minimal perfect map ([2]). In [1], it is shown that $Q(\mathscr{X})$ is a dense, point-separating subalgebra of $D(m\beta\mathscr{X})$, the set of all continuous maps from $m\beta\mathscr{X}$ into the two-point compactification of the real line which are real-valued on a dense subset of $m\beta\mathscr{X}$ (see, also, [3]). Since $Q(\mathscr{X})$ contains every ring of quotients of $C(\mathscr{X})$, this leads to

PROPOSITION 1. *If B is any ring of quotients of $C(\mathscr{X})$, then there exist a compact (Hausdorff) space $\mathscr{Y}$ and minimal perfect maps α and γ such that B is a point-separating subalgebra of $D(\mathscr{Y})$ and the following diagram commutes:*

$$\begin{array}{ccc} m\beta\mathscr{X} & \xrightarrow{\alpha} & \mathscr{Y} \\ & \searrow^{\tau} & \downarrow{\scriptstyle\gamma} \\ & & \beta\mathscr{X}\,. \end{array}$$