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MONOTONE DECOMPOSITIONS OF IRREDUCIBLE HAUSDORFF CONTINUA

G. R. GORDH, JR.

It is shown that a number of important results concerning irreducible metric continua can be generalized to (nonmetric) irreducible continua. For example, if M is a (nonmetric) continuum which is irreducible between a pair of points and which contains no indecomposable subcontinuum with interior, then there exists a monotone continuous map of M onto a generalized arc, such that each point inverse has void interior. This result is applied to a study of hereditarily unicoherent, hereditarily decomposable continua. Certain properties of trees follow as corollaries. Also, trees are characterized as inverse limits of monotone inverse systems of dendrites.

In recent years there has been a growing interest in the study of (nonmetric) continua. It is well known (e.g., [6]) that some of the most useful and important properties of metric continua do not hold for (nonmetric) continua. It is the purpose of this paper to indicate that a substantial number of theorems concerning irreducible metric continua can be generalized to irreducible continua. These results are then applied to a study of certain hereditarily unicoherent continua.

In particular, § 2 contains generalizations of many of the results about irreducible metric continua appearing in Chapter 1 of [11]. These results are applied in § 3 to obtain generalizations of a number of theorems due to Miller [8] concerning hereditarily unicoherent continua. Section 4 contains several results about trees which follow as corollaries of theorems in § 3. Also, it is proved that every tree can be written as a monotone inverse limit of dendrites. In Chapter 2 of [11], Thomas discusses metric continua which are hereditarily of type A'. His definition is extended, in § 5, to (nonmetric) continua and several characterizations of such continua are obtained.

The reader is referred to [3], [5], and [14] for general results concerning continua (i.e., compact, connected Hausdorff spaces). It will be necessary to refer to results which are stated in the literature for metric continua; however, this will be done only when the proof for continua is essentially the same as that for metric continua.

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