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COMPACT SEMIGROUPS WITH SQUARE ROOTS

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Suppose that S is a finite dimensional cancellative commutative clan with $E = \{0, 1\}$ and that H is the group of units of S. We show that if square roots exist in S/H, not necessarily uniquely, then there is a closed positive cone T in E^n for some n and a homomorphism $f: (T \cup \infty) \times H \to S$ which is onto and one-to-one on some neighborhood of the identity. $T \cup \infty$ denotes the one point compactification of T.

K. Keimel proved in (6), and Brown and Friedberg independently in (1), that if S/H is uniquely divisible, then it is isomorphic to $T \cup \infty$ for some closed positive cone T. Brown and Friedberg went on to show that if S is uniquely divisible, then S is isomorphic to the Rees quotient $((T \cup \infty) \times H)/(\infty \times H)$. What we do here is to weaken their hypothesis to assume just square roots in S/H and conclude that S is isomorphic to some quotient of such $(T \cup \infty) \times H$, which will be a Rees quotient if square roots are unique in $(S/H)\backslash 0$, but in general need not be Rees.[1] $f((T \cup \infty) \times 1)$ is a subclan of S and a local cross section at 1 for the orbits of the group action $H \times S \to S$ (which equal $\mathscr{H}$ classes here), but an example shows that it need not be a full cross section. Also, square roots exist (uniquely) in S if and only if they exist (uniquely) in S/H and H.

The proof consists essentially of showing that the ingenious constructions of (1) can still be done under the weaker hypothesis, in a sufficiently small neighborhood of H.

For basic information about semigroups, see (5), (8) or (9). The real intervals $(0, 1]$ and $[0, 1]$ are semigroups under usual real multiplication; as in (5), a *one parameter semigroup* is a homomorph of $(0, 1]$, and we also define here a *closed one parameter semigroup* to be a nonconstant homomorph of $[0, 1]$.

The Lemmas (I)-(III) are variations on standard themes so we omit proofs. (See (1), (3), (4), B-3 of (5), (6) and (7).) Throughout this paper let S be a clan with exactly two idempotents, a zero and an identity denoted by 0 and 1 respectively.

(I) *If R is a one parameter semigroup in S which is not contained in H and is not equal to 0, then $R \cup 0$ is a closed one parameter semigroup and an arc with endpoints 0 and 1. Let $\phi: (0, 1] \to R$ be the homomorphism that defines R; if $x = \phi(t) \in R$ and $k \geqq 0$, we write*

[1] Keimel has concurrently proved a further generalization, by a different method, assuming instead of cancellation that $x \times H \to xH$ is one-to-one for all x near H.