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ON THE OPEN CONTINUOUS IMAGES OF PARACOMPACT ČECH COMPLETE SPACES

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This article characterizes the completely regular T_0 open continuous images of paracompact Čech complete spaces. The characterization involves three conditions equivalent to being such an image. The first is an intrinsic condition concerning the position of the space in any of its Hausdorff bicompactifications. This condition weakens the condition of Čech completeness by replacing the concept of *G_δ-set* by that of *set of interior condensation*. This replacement yields a notion of topological completeness which has certain advantages over Čech completeness and uniform completeness but which reduces to Čech completeness in the case of metrizable spaces. The second condition (Condition $\mathscr{K}$) is intrinsically defined with the use of a sequence of collections of open sets. It is an analogue of the notion of a regular T_0-space having a monotonically complete base of countable order. The third condition is that of being an open continuous image of a space which is the sum of open Čech complete subspaces. The main theorem thus displays four equivalent forms of a topological completeness property invariant under open continuous mappings between Tychonoff spaces.

The characterization mentioned (Theorem 4) complements the characterization [15] of the Hausdorff open continuous images of T_2 paracompact p-spaces as Hausdorff spaces of point-countable type in a way analogous to that in which the characterization [14] of regular T_0 open continuous images of complete metric spaces as regular T_0-spaces having monotonically complete bases of countable order complements Ponomarev's characterization [11] of the T_0 open continuous images of metrizable spaces as the T_0 first countable spaces. It is relevant to recall in this connection some results of Frolík [7] and Arhangel'skiĭ [4], respectively: The class of T_2 paracompact Čech complete spaces (T_2 paracompact p-spaces) is the class of T_2 perfect preimages of complete metric spaces (metric spaces).

In [13] it was shown that a Tychonoff open continuous image of a paracompact Čech complete space (in fact, of a metrically topologically complete space) is not necessarily Čech complete. This affords interesting contrast with the behavior guaranteed by Theorem 5: A Tychonoff open continuous image of a Tychonoff space complete in the sense of Condition $\mathscr{K}$ is also complete in the same sense.

Some results of the paper have antecedents in the classical theory