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THE HAUSDORFF MEANS OF DOUBLE FOURIER SERIES AND THE PRINCIPLE OF LOCALIZATION

FRED USTINA

In the two dimensional case, as in the one dimensional case, the Hausdorff summability method is generated by a Hausdorff weight function. In this paper, we investigate the conditions which must be imposed on this weight function in order that the resulting means of a double Fourier series will display the principle of localization.

In this article we examine the conditions under which the Hausdorff means of double Fourier series exhibit the principle of localization. As is well known, these means are a generalization of a number of other well known means, including those of Cesáro and Euler. Our results are summarized in Theorems 1 to 4, together with the appropriate corollaries.

Let $[c, d; a, b]$ denote a rectangle with vertices at (a, b), (a, d), (c, b) and (c, d), $a \leqq c$, $b \leqq d$. For $0 < \delta < \pi$, let $R(\delta) = [\delta, \delta; -\delta, -\delta]$, $N(\delta) = [\pi, \delta; -\pi, -\delta] \cup [\delta, \pi; -\delta, -\pi]$, $C(\delta) = [\pi, \pi; -\pi, -\pi] \sim N(\delta)$, and $E(\delta) = N(\delta) \sim R(\delta)$. For $0 < \tau \leqq 1/2$, let $\Delta(\tau) = [1-\tau, 1-\tau; \tau, \tau]$, and let $\theta(\tau) = [1, 1; 0, 0] \sim \Delta(\tau)$. Then $N(\delta)$ is a cross-neighborhood of the origin, $E(\delta)$ is the deleted cross-neighborhood, and $\theta(\tau)$ is the τ-neighborhood of the boundary of the unit square $[1, 1; 0, 0]$.

Let $f(x, y)$ be a 2π-periodic function, Lebesgue integrable in the period square, and let $\{s_{mn}(x, y)\}$ be the corresponding sequence of partial sums of the Fourier series of $f(x, y)$. In the sequel we relate all such calculations to the origin, so that we will be examining the sequence $\{s_{mn}(0, 0)\}$, which we denote simply by $\{s_{mn}\}$. As is easily shown,

$$\begin{aligned} s_{mn} &= \frac{1}{4\pi^2}\int_{-\pi,-\pi}^{\pi,\pi} f(s, t)\,\frac{\sin(m+1/2)s}{\sin s/2}\,\frac{\sin(n+1/2)t}{\sin t/2}\,ds\,dt \\ &= \frac{1}{4\pi^2}\left\{\int_{R(\delta)} + \int_{E(\delta)} + \int_{C(\delta)}\right\} \qquad (1) \\ &= r_{mn} + e_{mn} + c_{mn}\,. \end{aligned}$$

Now suppose that a regular linear summability method H ([6], Vol. 1, p. 74) is applied to the sequence $\{s_{mn}\}$ and let $\{h_{mn}\}$ denote the corresponding sequence of transforms under the method H. Then

$$\begin{aligned} h_{mn} &= H\{s_{mn}\} \\ &= H\{r_{mn}\} + H\{e_{mn}\} + H\{c_{mn}\} \qquad (2) \\ &= \alpha_{mn} + \beta_{mn} + \gamma_{mn}\,. \end{aligned}$$