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TORSION THEORIES AND RINGS OF QUOTIENTS OF MORITA EQUIVALENT RINGS

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A ring of left quotients $Q_{\mathscr{T}}$ of a ring R can be constructed relative to any hereditary torsion class $\mathscr{T}$ of left R-modules. For Morita equivalent rings R and S we construct a one-to-one correspondence between the hereditary torsion classes (strongly complete Serre classes) of ${}_R\mathfrak{M}$ and ${}_S\mathfrak{M}$ and describe the resulting correspondence between the strongly complete filters of left ideals of R and S. We show that the proper rings of left quotients of R and S relative to corresponding hereditary torsion classes are Morita equivalent. Applications are made to the maximal and the classical rings of left quotients and the corresponding torsion theories.

A *torsion theory* for the category ${}_R\mathfrak{M}$ of unitary left modules over an associative ring R with identity has been defined by Dickson [3] to be a pair $(\mathscr{T}, \mathscr{F})$ of classes of left R-modules such that

(a) $\mathscr{T} \cap \mathscr{F} = \{0\}$

(b) $\mathscr{T}$ is closed under homomorphic images

(c) $\mathscr{F}$ is closed under submodules

(d) for every left R-module M there exists a submodule $T(M)$ of M with $T(M) \in T$ and $M/T(M) \in \mathscr{F}$.

A class $\mathscr{T}(\mathscr{F})$ of left modules is called a *torsion* (*torsion-free*) class if there is a (necessarily unique) class $\mathscr{F}(\mathscr{T})$ such that $(\mathscr{T}, \mathscr{F})$ is a torsion theory. A torsion class closed under submodules is said to be *hereditary*. By [3, Theorem 2.3] a class $\mathscr{T}$ is a hereditary torsion class if and only if it is closed under submodules, homomorphic images, extensions, and arbitrary direct sums. Walker and Walker [13] call such a class a *strongly complete Serre class*. Gabriel [4] has shown that for a ring R there is a one-to-one correspondence between the strongly complete Serre classes of ${}_R\mathfrak{M}$ and the strongly complete filters F of left ideals of R given by the mapping

$$\mathscr{T} \longrightarrow F(\mathscr{T}) = \{I \leqq R \mid R/I \in \mathscr{T}\}$$

where $I \leqq R$ denotes that I is a left ideal of R. The inverse correspondence is given by

$$F \longrightarrow \mathscr{T}(F) = \{M \in {}_R\mathfrak{M} \mid (0\colon m) \in F \text{ for all } m \in M\}$$

where $(0\colon m) = \{r \in R \mid rm = 0\}$. We say a strongly complete filter F of left ideals of R is *faithful* if $(0\colon r) \in F$ implies $r = 0$ for each $r \in R$. A strongly complete Serre class $\mathscr{T}$ is called a *faithful Serre*