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STRUCTURE OF NOETHER LATTICES WITH JOIN-PRINCIPAL MAXIMAL ELEMENTS

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In this paper we explore the structure of Noether lattices with join-principal maximal elements.

Results which completely specify the structure of certain special classes of Noether lattices, and relate them to lattices of ideals of Noetherian rings, have been obtained in [1], [2], [3], [4], [7], and [8]. For example, in [7] we showed that if every maximal element of a Noether lattice $\mathscr{L}$ is meet-principal, then $\mathscr{L}$ is distributive and can be represented as the lattice of ideals of a Noetherian ring. Moreover, for distributive Noether lattices, the condition that every maximal element is meet-principal is equivalent to representability. In a more recent paper [8], we began considering the complementary case of a Noether lattice in which every maximal element is join-principal in order to determine the extent of the relationship between the two situations. There we showed that if 0 is prime in $\mathscr{L}$ (and every maximal element is join-principal), then $\mathscr{L}$ is distributive and representable. Hence, if 0 is prime, the assumptions that every maximal element is meet-principal and that every maximal element is join-principal are equivalent, and either implies representability.

In this paper, we continue the investigation begun in [8]. Our results extend the class of Noether lattices for which embedding and structure theorems are known, and also introduce a construction process for Noether lattices which leads to new examples.

In § 1, we show that in a local Noether lattice $(\mathscr{L}, M)$ in which M is join-principal and not a prime of 0, the maximal element M has a minimal base $E_1, \cdots, E_k$ of independent principal elements (i.e., $E_i \wedge (E_1 \vee \cdots \vee \hat{E}_i \vee \cdots \vee E_k) = 0$ for $i = 1, \cdots, k$). And we use this result to show that if M is join-principal and not a prime of 0, then $\mathscr{L}$ is distributive. In § 2, we obtain structure and embedding theorems for distributive local Noether lattices with join-principal maximal elements. In § 3, we investigate some of the consequences of our results outside of the local case.

We adopt the terminology of [5].

1. Let $(\mathscr{L}, M)$ be a local Noether lattice and let $B \in \mathscr{L}$. The quotient B/MB is a finite dimensional complemented modular lattice and the number of elements in any minimal set of principal elements with join B is the dimension of the quotient B/MB ([4], [6]). Hence