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MULTIPLIERS AND UNCONDITIONAL CONVERGENCE OF BIORTHOGONAL EXPANSIONS

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We solve in the affirmative a problem raised by B. S. Mityagin in 1961, namely, we prove that if (x_n, f_n) is a biorthogonal system for a Banach space E with (f_n) total over E, such that the set of multipliers $M(E, (x_n, f_n))$ contains all sequences (ε_i) with $\varepsilon_i = \pm 1$ for each i, then (x_n) is an unconditional basis for E.

Let E be a Banach space, and let (x_n, f_n) be a biorthogonal system for E (i.e., $(x_n) \subset E$, $(f_n) \subset E^*$ and $f_n(x_m) = \delta_{nm}$) which has (f_n) total over E (i.e., $f_n(x) = 0$ for all n implies $x = 0$). A scalar sequence (γ_n) is called a *multiplier* of an element x in E with respect to (x_n, f_n) (write $(\gamma_n) \in M(x, (x_n, f_n))$) if there is an element y of E such that $f_n(y) = \gamma_n f_n(x)$ for all n (call this element $x_{(\gamma_n)}$). The set of multipliers for E with respect to (x_n, f_n) is

$$M(E, (x_n, f_n)) = \cap \{M(x, (x_n, f_n)) \mid x \in E\} .$$

Here we consider the following two problems:

P 1: (Mityagin [6], Kadec-Pelczynski [4], Pelczynski [7]). Let E be separable and suppose that $M(E, (x_n, f_n))$ contains all sequences (ε_i) with $\varepsilon_i = \pm 1$ for each i. Is (x_n) an unconditional basis for E?

P 2: (Kadec-Pelczynski [4]). Let E be separable and suppose $M(x, (x_n, f_n))$ contains all sequences (ε_i) with $\varepsilon_i = \pm 1$ for each i. Does the formal expansion $\sum_n f_n(x) x_n$ converge unconditionally to x?

Problem 2 (and hence also problem 1) is known to have an affirmative answer in the following cases [4]:

1°. $M(x, (x_n, f_n)) \supset m$ (the space of bounded sequences).

2°. E contains no subspace isomorphic to c_0 (the space of sequences converging to 0) and $M(x, (x_n, f_n)) \supset c_0$.

3°. $sp(f_n)$ (= linear span of (f_n)) is norming (i.e.,

$$|x| = \sup \{|f(x)| \mid f \in sp(f_n), \|f\| \leqq 1\}$$

defines a norm on E equivalent to the original norm on E).

Problem 1 is known to have an affirmative answer in the case when $[x_n] = E$, where $[x_n]$ denotes the closed linear span of $\{x_n\}$ ([5]; see also [1], Theorem 3.4, implication (4) $\Rightarrow$ (3)).

In the present paper we give an affirmative solution for problem