

CO-ABSOLUTES OF REMAINDERS OF STONE-CECH COMPACTIFICATIONS

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Let X be a completely regular Hausdorff space. Denote the "absolute" (also called the "projective cover") of X by $E(X)$, the Boolean algebra of regular closed subsets of X by $R(X)$, and the Stone-Cech compactification of X by βX . In this paper it is proved that the canonical map $k: E(\beta X) \rightarrow \beta X$ maps $\beta E(X) - E(X)$ irreducibly onto $\beta X - X$ if and only if the map $A \rightarrow cl_{\beta X} A - X$ is a Boolean algebra homomorphism from $R(X)$ into $R(\beta X - X)$. This latter condition is shown to hold for a wide class of spaces X . These results are used to calculate absolutes and well-known co-absolutes of $\beta X - X$ under several different sets of hypotheses concerning the topology of X .

Throughout this paper we use without further comment the notation and terminology of the Gillman-Jerison text [6]. In particular, the cardinality of a set S is denoted by $|S|$. The countable discrete space is denoted by N , and the set of nonnegative integers (used as an index set) is denoted by N . The symbol $[CH]$ appearing before the statement of a theorem indicates that the continuum hypothesis ($\aleph_1 = 2^{\aleph_0}$) is used in the proof of the theorem. The cardinal 2^{\aleph_0} will be denoted by the letter c . All topological spaces considered in this paper are assumed to be completely regular Hausdorff spaces. This assumption is repeated for emphasis from time to time.

In § 1 we give a brief summary of known results and define some notation and terminology. Some of the results in later sections are generalizations of results appearing in [17]. Background material on Boolean algebras appears in [15].

1. Preliminaries. The concept of the absolute of a topological space has been considered by several authors, notably Gleason [7], Iliadis [8], Flachsmeyer [5], Ponomarev [12], and Strauss [16]. In the first part of this section we give a brief outline of this theory. Although a theory of absolutes can be developed for a wider class of topological spaces, we shall assume that all spaces considered are completely regular and Hausdorff.

Recall that a subset A of a topological space X is said to be *regular closed* if $A = cl_X(int_X A)$. Let $R(X)$ denote the family of all regular closed subsets of X . The following theorem is well-known; see, for example, § 1 and § 20 of [15].