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GAUSSIAN MARKOV EXPECTATIONS AND RELATED INTEGRAL EQUATIONS*

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Let $\{X(w), s \leqq w \leqq t\}$ be a Gaussian Markov stochastic process with continuous sample functions. Examples of such processes are the Wiener, Ornstein-Uhlenbeck, and Doob-Kac processes. An operator valued function space integral is defined for each process. This was done for the Wiener process by R. H. Cameron and D. A. Storvick. For functionals of the form $F(x) = \exp\left\{\int_s^t \theta(t-w, x(w))dw\right\}$ where $\theta(t, u)$ is bounded and almost everywhere continuous, the special integrals satisfy integral equations related to the generalized Schroedinger equations studied by the first author. For the Wiener process, a "backwards time" equation is coupled with the Cameron-Storvick equation to give a pair of integral equations.

In [12] R. H. Cameron and D. A. Storvick defined an operator valued function space integral based on the Wiener stochastic process. For an appropriate functional, such an integral solves an integral equation related to the Schroedinger equation. The purpose of this paper is to define such integrals for Gaussian Markov stochastic processes, and prove that for appropriate functionals they satisfy an integral equation related to the generalized Schroedinger equation discussed by the first author in [5], [6], [7], and [8]. Examples of Gaussian Markov processes are the Wiener, Ornstein-Uhlenbeck, and Doob-Kac processes. For the Wiener process we will give a "backwards time" equation which when coupled with the Cameron-Storvick "forwards time" equation will give a *pair* of integral equations. That a function space integral solves a pair of integral equations was first done in [14] by D. A. Darling and A. J. F. Siegert.

This area of research is motivated, in many respects, by R. P. Feynman's function space integral which he first discussed in 1948 [16]. Since then extensive work has been done to enlarge the class of functionals for which "Feynman integrals" exist. See, for example, the work of R. H. Cameron [9, 10, 11], Donald Babbitt [1, 2, 3, 4], Jacob Feldman [15], K. Itô [18, 19], Edward Nelson [22], and G. W. Johnson and D. L. Skoug [20, 21, 23]. In the papers by Cameron and Storvick [12, 13] the integral equation involved is related to the Schroedinger equation. A heuristic discussion of that relation is

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