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SEMI-DEVELOPABLE SPACES AND QUOTIENT IMAGES OF METRIC SPACES

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In this paper semi-developable spaces are defined and, among T_0-spaces, are shown to be the same as the semi-metrizable spaces. Strongly semi-developable spaces are defined in a natural way and proven to coincide with an important class of semi-metric spaces, namely those in which "Cauchy sequences suffice". These spaces are shown to possess several other interesting properties. Probably the most significant of these is that the strongly semi-developable spaces are the hereditarily quotient P-images of metric spaces. Other quotient images of metric spaces are similarly characterized in terms of semi-developments.

1. Semi-developable spaces. Until fairly recently there were almost no non-trivial topological restrictions on a space which guarantee the space to be semi-metrizable. Work along this line was initiated by Alexandrof and Niemytskii in their paper [1]. They proved that the developable spaces are precisely the spaces having a semi-metric satisfying the following condition. At each point there is a neighborhood of arbitrarily small diameter. Or equivalently, every convergent sequence is a Cauchy sequence. More recently Heath [10], Ceder [6], McAuley [13], and Arhangel'skii [3] have contributed new results in this area. Moreover, several authors have observed that there are numerous similarities between developable spaces and semi-metrizable spaces. This is especially true since so many theorems which hold for developable spaces are also valid for semi-metrizable spaces. In Theorem 1.3, the semi-metrizable spaces are shown to be the semi-developable T_0-spaces. This indicates the nature of the similarities between developable spaces and semi-metrizable spaces. Strongly semi-developable spaces are defined and, in Theorems 1.5 and 1.6, are shown to form a natural intermediate class of spaces between the semi-metrizable spaces and the developable spaces.

By a space we will mean a topological space as defined in [12]. All other definitions pertaining to topological spaces not specifically given in this paper are as found in [12].

DEFINITION. A *development* for a space X is a sequence

$$\Delta = \{g_n \mid n = 1, 2, \cdots\}$$

of open covers of X such that $\{St(x, g_n) \mid n = 1, 2, \cdots\}$ is a local base