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MODULAR ANNIHILATOR A*-ALGEBRAS

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This paper is concerned with modular annihilator A^*-algebras. Let A be an A^*-algebra, B a maximal commutative *-subalgebra of A and X_B the carrier space of B. We show that the following statements are equivalent: (i) A is a modular annihilator algebra. (ii) Every X_B is discrete. (iii) Every B is a modular annihilator algebra. (iv) The spectrum of every hermitian element of A has no nonzero limit points.

Let A be an A^*-algebra which is a dense two-sided ideal of a B^*-algebra $\mathfrak{A}$, A^{**} the second conjugate space of A and π_A the canonical embedding of A into A^{**}. We show that A is a modular annihilator algebra if and only if $\pi_A(A)$ is a two-sided ideal of A^{**} (with the Arens product). This generalizes a recent result by B. J. Tomiuk and the author.

The theory of (left, right) modular annihilator algebras was developed in [20]. In a recent paper [4], Barnes has extended this study to semi-simple Banach algebras. He has proved an interesting result which says that if A is a semi-simple Banach algebra, then A is modular annihilator if and only if the spectrum of every element of A has no nonzero limit points (see [4; p. 516, Theorem 4.2]). In this paper, we show that a similar result holds for A^*-algebras.

2. **Notation and preliminaries.** Notation and definitions not explicitly given are taken from Rickart's book [15].

For any subset E of a Banach algebra A, let $L_A(E)$ and $R_A(E)$ denote the left and right annihilators of E in A, respectively. Then A is called a modular annihilator algebra if, for every maximal modular left ideal I and for every maximal modular right ideal J, we have $R_A(I) = (0)$ if and only if $I = A$ and $L_A(J) = (0)$ if and only if $J = A$. Let A be a semi-simple modular annihilator Banach algebra. Then every left (right) ideal of A contains a minimal idempotent (see [2; p. 569, Theorem 4.2]).

A Banach algebra with an involution $x \rightarrow x^*$ is called a Banach *-algebra. A Banach *-algebra A is called a B^*-algebra if the norm and the involution satisfy the condition $\|x^*x\| = \|x\|^2$ $(x \in A)$. If A is a Banach *-algebra on which there is defined a second norm $|.|$, which satisfies, in addition to the multiplicative condition $|xy| \leqq |x|\,|y|$, the B^*-algebra condition $|x^*x| = |x|^2$, then A is called an A^*-algebra. The norm $|.|$ is called an auxiliary norm. Let A be an A^*-algebra. Then the involution $x \rightarrow x^*$ in A is continuous with respect to the given norm and the auxiliary norm and every closed *-subalgebra of