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A CONGRUENCE THEOREM FOR ASYMMETRIC TREES

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The question is studied how a given tree is determined by the collection of its asymmetric subtrees. The results are analogous other partial answers to the Ulam-Kelly conjecture.

In [1], [2], [4], [5] several theorems are proved concerning the following conjecture posed by P. J. Kelly [4]: If G and H are two graphs with p vertices v_i and u_i respectively ($p \geqq 3$) such that for all i: $G - v_i \cong H - u_i$ then G and H are themselves isomorphic. In [4] it is shown that this conjecture is true when G, H are trees. In [1], [2], [5] improvements of this result are obtained, namely, knowledge any of the following collections is sufficient to conclude $G \cong H$ providing G, H are trees:

(1) all maximal proper subtrees [2]

(2) subtrees $T - v_i$ where v_i is a peripheral vertex [1]

(3) non-isomorphic maximal subtrees [5].

Let $G(T)$ denote the automorphism group of a tree T. If $G(T) =$ {identity} then T is called an asymmetric tree. Let $\mathfrak{A}$ denote the class of all asymmetric trees.

For a tree T consider the set of all asymmetric proper subtrees of T. This set is naturally partially ordered by inclusion, denote by $A(T)$ the set of all maximal elements of this set, i.e. the set of all maximal asymmetric subtrees. (By subtree is meant proper subtree from now on.) Further denote by $\mathfrak{A}(T)$ the set of all isomorphism types of $A(T)$. (We denote by $[G]$ the isomorphism type of the graph G, hence $\mathfrak{A}(T) = \{[T']: T' \in A(T)\}$.) We write $A(T) \cong A(S)$ for trees T and S, if there is a one-to-one mapping $\varphi: A(T) \rightarrow A(S)$ such that $\varphi(T_i) \cong T_i$ for every $T_i \in A(T)$.

We write $\mathfrak{A}(T) = \mathfrak{A}(S)$ if the sets $\mathfrak{A}(T)$ and $\mathfrak{A}(S)$ are equal. We write $T_{i,j,k}$ for the tree consisting of three edge disjoint paths that start from a common point and have lengths i, j, k.

We will investigate the dependence of $[T]$ on $A(T)$ and $\mathfrak{A}(T)$.

It is obvious that not every tree T will be determined by $A(T)$, since there are nonisomorhic trees with $A(T) = \emptyset$ (we do not include the trivial tree in the collections $A(T)$ and $\mathfrak{A}(T)$). But such trees are characterized by the following known result:

PROPOSITION 0.1. *We have $A(T) \neq \emptyset$ iff $T_7 < T$, where $T_7 = T_{1,2,3}$ with 7 vertices is the minimal asymmetric tree and $G < H$ means that G is a proper full subgraph of H.*