PACIFIC JOURNAL OF MATHEMATICS
Vol. 37, No. 3, 1971

STRATIFIABLE SPACES, SEMI-STRATIFIABLE SPACES, AND THEIR RELATION THROUGH MAPPINGS

MICHAEL HENRY

It is shown that the image of a stratifiable space under a pseudo-open compact mapping is semi-stratifiable. By strengthening the mapping from compact to finite-to-one the following results are also obtained. The image of a semi-stratifiable (semi-metric) space under an open finite-to-one mapping is semi-stratifiable (semi-metric).

Notation and terminology will follow that of Dugundji [6]. By a neighborhood of a set A, we will mean an open set containing A, and all mappings will be continuous and surjective.

DEFINITION 1.1. A topological space X is a *stratifiable space* if, to each open set $U \subset X$, one can assign a sequence $\{U_n\}_{n=1}^{\infty}$ of open subsets of X such that

(a) $\bar{U}_n \subset U$,
(b) $\bigcup_{n=1}^{\infty} U_n = U$,
(c) $U_n \subset V_n$ whenever $U \subset V$.

DEFINITION 1.2. A topological space X is a *semi-stratifiable space* if, to each open set $U \subset X$, one can assign a sequence $\{U_n\}_{n=1}^{\infty}$ of closed subsets of X such that

(a) $\bigcup_{n=1}^{\infty} U_n = U$,
(b) $U_n \subset V_n$ whenever $U \subset V$.

Ceder [3] introduced M_3-spaces and Borges [2] renamed them "stratifiable", while Creede [4] studied semi-stratifiable spaces. A correspondence $U \rightarrow \{U_n\}_{n=1}^{\infty}$ is a *stratification* (semi-stratification) for the space X whenever it satisfies the conditions of Definition 1.1 (1.2).

LEMMA 1.3. *A space X is stratifiable if and only if to each closed subset $F \subset X$ one can assign a sequence $\{U_n\}$ of open subsets of X such that*

(a) *$F \subset U_n$ for each n,*
(b) *$\bigcap_{n=1}^{\infty} \bar{U}_n = F$,*
(c) *$U_n \subset V_n$ whenever $U \subset V$.*

LEMMA 1.4. *A space X is semi-stratifiable if and only if to each closed set $F \subset X$ one can assign a sequence $\{U_n\}$ of open subsets*