PACIFIC JOURNAL OF MATHEMATICS
Vol. 37, No. 3, 1971

PRIME GENERATORS WITH PARABOLIC LIMITS

JOHN HARRIS AND OLGA HIGGINS

The prime generating properties of the formula

$$F = \frac{AX^2 + ABXY + CY^2}{(A, Y)}, \quad (X, Y) = 1$$

are developed by way of three theorems. Theorem I is a prime test for F, Theorem II will factor a composite F, and Theorem III establishes parabolic limits; within these limits F is always prime.

In the 18th century Leonhard Euler and A. M. Legendre found several "prime generating" polynomials. Euler's famous formula $X^2 + X + 41$ takes prime values for every integral value of x from 0 to 39, and Legendre's formula $2x^2 + 29$ does almost as well, taking prime values for every integral value of x from 0 to 28. These and many other expressions that have been found since have coefficients of the form $[A, AB, C]$, with $B = 0$ or 1 and C a prime.

After numerous experiments with two variables we have chosen

$$F = \frac{AX^2 + ABXY + CY^2}{E}, \quad E = (A, Y), (X, Y) = 1$$

as our basic "prime generating" formula. The coefficients A, B and

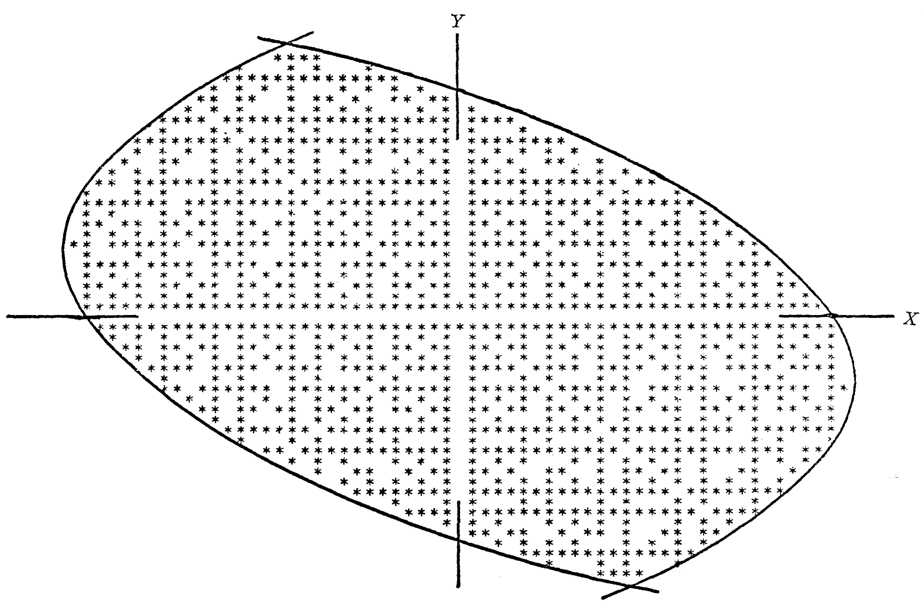

FIGURE 1