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C-COMPACT AND FUNCTIONALLY COMPACT SPACES

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In the first section of this note a question posed by G. Viglino is resolved by constructing a C-compact space which is not seminormal. In the second section some characterizations of C-compact and functionally compact spaces are introduced. In the final section, embedding theorems of spaces into C-compact and functionally compact spaces are noted.

1. A Counterexample.

DEFINITIONS. (a) A Hausdorff space X is *absolutely closed* if given an open cover $\mathscr{V}$ of X, then there exists a finite number of elements of $\mathscr{V}$, say V_i, $1 \leqq i \leqq n$, with $X \subset \mathrm{Cl} \bigcup_{i=1}^{n} V_i$.

(b) A Hausdorff space (X, τ) is *C-compact* if given a closed set Q of X and a τ-open cover $\mathscr{V}$ of Q, then there exists a finite number of elements of $\mathscr{V}$, say V_i, $1 \leqq i \leqq n$, with $Q \subset Cl_X \bigcup_{i=1}^{n} V_i$.

(c) An open set V is *regular* if $V = \bar{V}^{\circ}$.

(d) A space X is *seminormal* if given a closed subset C of X and an open set V containing G, then there exists a regular open set R with $C \subset R \subset V$.

G. Viglino has shown that a seminormal absolutely closed space is C-compact, and posed the question as to whether or not the converse holds [5]. The following is an example of a C-compact space which is not seminormal. An example has also been obtained by T. Lominac, Abstract #682-54-33.

EXAMPLE. Let Z represent the set of positive integers. Let

$$X = \left\{\left(\frac{1}{2n-1}, \frac{1}{m}\right) \middle| n, m \in Z\right\} \cup \left\{\left(\frac{1}{2n}, -\frac{1}{m}\right) \middle| n, m \in Z\right\} \cup \left\{\left(\frac{1}{n}, 0\right) \middle| n \in Z\right\} \cup \{\infty\} .$$

Topologize X as follows. Partition Z into infinitely many infinite equivalence classes, $\{Z_i\}_{i=1}^{\infty}$, and let $\{Z_1^J\}_{J=1}^{\infty}$ be a partition of Z_1 into infinitely many infinite equivalent classes. Let Φ denote a bijection from $\{(1/(2n-1), 1/m) \mid n, m \in Z\}$ to $Z \backslash \{1\}$. Let a neighborhood system for the points of the form $(1/(2i-1), 0)$ be composed of all sets of the form $U_{(2i-1,0;k)} = V \cup F$ where $V = \{(1/(2i-1), 0)\} \cup \{(1/(2i-1), 1/m) \mid m \geqq k\}$ and $F = \{(1/(2n-1), 1/m) \mid m \in Z_i \text{ and } n \geqq k\} \cup \{(1/2n, -1/s) \mid$