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ANALYTIC SHEAVES ON KLEIN SURFACES

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Morphisms of Klein surfaces are discussed from the sheaf-theoretic standpoint, and the cohomology of an analytic sheaf on a Klein surface is computed.

0. Let $\mathfrak{X}$ be a Klein surface [1], [2]; that is, $\mathfrak{X}$ consists of an underlying space X, which is a surface with boundary, and a family of equivalent dianalytic atlases on X. If (U_α, z_α) is such an atlas, then $z_\alpha\colon U_\alpha \to \boldsymbol{C}^+$ is a homeomorphism of the open set U_α in X onto an open subset of $\boldsymbol{C}^+ = \{z \in \boldsymbol{C} \mid \operatorname{Im}(z) \geqq 0\}$. The functions z_α must thus be real on $U_\alpha \cap \partial X$, and it is required that $z_\alpha \circ z_\beta^{-1}$ be dianalytic, that is, either analytic or antianalytic on each component of $z_\beta(U_\alpha \cap U_\beta)$.

In this paper we define the structure sheaf of $\mathfrak{X}$, show that the concept of morphism given in [1], [2] coincides with the concept of a morphism of ringed spaces, and compute the cohomology of analytic sheaves on $\mathfrak{X}$. If $\mathscr{F}$ is an analytic sheaf on X, and $\tilde{\mathscr{F}}$ is the lift of $\mathscr{F}$ to the complex double $\tilde{\mathfrak{X}}$ of $\mathfrak{X}$, then there is a natural isomorphism

$$H^q(\tilde{\mathfrak{X}}, \tilde{\mathscr{F}}) \cong \boldsymbol{C} \otimes_{\boldsymbol{R}} H^q(\mathfrak{X}, \mathscr{F}) .$$

1. **The structure sheaf $\mathscr{O}_{\mathfrak{X}}$.** We define the structure sheaf $\mathscr{O}_{\mathfrak{X}} = \mathscr{O}$ on $\mathfrak{X}$ as follows. If U is open in X, let $\mathscr{O}(U)$ be the ring of holomorphic functions on U (in the sense of [1], [2]). If $U \supset U'$, then the inclusion map is a morphism of Klein surfaces and we have a natural map $\rho^U_{U'}\colon \mathscr{O}(U) \to \mathscr{O}(U')$ (this is not quite an ordinary restriction map since the elements of $\mathscr{O}(U)$ are not quite functions). In particular, if (U_α, z_α) and (U_β, z_β) are dianalytic charts on $\mathfrak{X}$, $U_\alpha \supset U_\beta$, then

$$\mathscr{O}(U_\alpha) \cong \left\{ \begin{matrix} f\colon U_\alpha \to C \mid f(U_\alpha \cap \partial X) \subset \boldsymbol{R}\} , \\ \text{and } f \circ z_\alpha^{-1} \text{ analytic} \end{matrix} \right\}$$

and

$$\rho^{U_\alpha}_{U_\beta}(f) = \begin{cases} f \mid U_\beta \text{ where } z_\alpha \circ z_\beta^{-1} \text{ is analytic} \\ \bar{f} \mid U_\beta \text{ where } z_\alpha \circ z_\beta^{-1} \text{ is antianalytic} . \end{cases}$$

It is easily checked that this defines a sheaf of local R-algebras on $\mathfrak{X}$.

Let $\mathfrak{X}, \mathfrak{Y}$ be Klein surfaces, $f\colon \mathfrak{Y} \to \mathfrak{X}$ a continuous map. Then f is a morphism [1] if $f(\partial Y) \subset \partial X$ and if for every point $p \in Y$ there