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THE COHOMOLOGY OF DIVISORIAL VARIETIES

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A well known theorem of Serre states the equivalence between the ampleness of a linear equivalence class of divisors on an algebraic variety and the vanishing of the first cohomology groups related to sufficiently high multiples of such linear equivalence class. In this paper the result of the above theorem is extended in the following direction: given a linear equivalence class on an algebraic variety, does there exist a cohomological characterization of the open subset consisting of points of the variety which belong to affine open complements of effective divisors in the multiples of the given class? The characterization obtained is the main result, and it gives easily Serre's result as a particular case. While in one direction the proof uses the vanishing theorem quoted in the beginning, it is independent of it in the opposite direction. A simple application of the main result gives a first cohomological characterization of divisorial varieties.

We prove first two lemmas, which hold, in our opinion, an intrinsic value, then we prove our main result, namely Theorem 4.

We use throughout this paper the notations and language of [6] and [8]. We deal entirely with schemes of finite type over an algebraically closed groundfield k, and we refer to them, for brevity's sake, simply as schemes. We shall also say "proper schemes" rather than schemes proper over Spec (k).

When we refer to, say, Lemma 3, without any further reference, we mean Lemma 3 of the present work.

We begin with the two lemmas mentioned in the introduction, needed later in the proof of Theorem 4.

LEMMA 1. *Let $f: X \to Y$ be a projective morphism of schemes. Let $\mathscr{L}$ be an invertible sheaf over X, ample for f, and let*

$$\mathscr{G}' \longrightarrow \mathscr{G} \longrightarrow \mathscr{G}'' \longrightarrow 0$$

be an exact sequence of coherent sheaves over Y. Then, for $n \gg 0$, the sequence of coherent sheaves over Y

$$\begin{aligned} f_*(\mathscr{L}^{\otimes n} \otimes f^*(\mathscr{G}')) &\longrightarrow f_*(\mathscr{L}^{\otimes n} \otimes f^*(\mathscr{G})) \\ &\longrightarrow f_*(\mathscr{L}^{\otimes n} \otimes f^*(\mathscr{G}'')) \longrightarrow 0 \end{aligned}$$

is exact.