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FUNCTIONALLY COMPACT SPACES, C-COMPACT SPACES AND MAPPINGS OF MINIMAL HAUSDORFF SPACES

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Our interest in this paper is in the mapping properties of minimal Hausdorff spaces; some of the results will provide new characterizations of the classes of functionally compact and *C*-compact spaces. Of more than secondary interest, it may be the primary message of the paper, is the point of view adopted (and outlined in § 2) in studying the "divisibility" of the highly nondivisible class of minimal Hausdorff spaces.

1. Introduction. Let X be a Hausdorff space. Then X is *absolutely closed* (AC) iff whenever X is embedded in a Hausdorff space Y, X is closed in Y. We call X *minimal Hausdorff* (MH) iff X admits no one-to-one continuous map to a Hausdorff space which is not a homeomorphism. X is *functionally compact* (FC) iff every continuous map on X to a Hausdorff space is a closed map. Finally, Velicko [13] has defined a set A in a space X to be an *H-set* iff for each family of sets open in X and covering A, there is a finite subfamily whose closures in X cover A. Porter and Thomas [11; Thm. 2.5] have observed that in Hausdorff spaces H-sets are closed, and Viglino [14] has defined a Hausdorff space to be *C-compact (CC)* iff every closed set is an H-set.

Some of the basic results we will need concerning the classes of spaces defined above are given in the following theorem.

THEOREM 1.1. *Let X be a Hausdorff space. Then*

(a) ([4]) *X is AC iff every open filter on X has a cluster point,*

(b) ([4]) *X is* MH *iff every open filter on X with a unique cluster point converges* (necessarily to that point),

(c) ([5]) *X is* FC *iff whenever $\mathscr{U}$ is an open filter base on X such that $\cap \{U \mid U \in \mathscr{U}\} = \cap \{\bar{U} \mid U \in \mathscr{U}\}$, then $\mathscr{U}$ is a base for the neighborhoods of $\cap \{\bar{U} \mid U \in \mathscr{U}\}$.*

(d) ([15]) *X is* CC *iff every open filter base $\mathscr{U}$ on X is a base for the nhoods of $\cap \{\bar{U} \mid U \in \mathscr{U}\}$.*

Each of the characteristic properties above can be applied to non-Hausdorff spaces. For example, a (not necessarily Hausdorff) space X is *generalized minimal Hausdorff* (GMH) iff every open filter with a unique cluster point converges. Similar definitions can be given for *generalized absolutely closed* (GAC), *generalized functionally compact* (GFC) and *generalized C-compact* (GCC) spaces.