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WHITTAKER CONSTANTS FOR ENTIRE FUNCTIONS OF SEVERAL COMPLEX VARIABLES

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Let f be an entire function of a single complex variable. The *exponential type* of f is given by

$$\tau(f) = \limsup_{n\to\infty} | f^{(n)}(0) |^{1/n} .$$

The Whittaker constant W is defined to be the supremum of numbers c having the following property: if $\tau(f) < c$ and each of $f, f', f'', \cdots$ has a zero in the disc $|z| \leqq 1$, then $f \equiv 0$. The Whittaker constant is known to lie between .7259 and .7378.

The present paper provides a definition and characterization of the Whittaker constant $\mathscr{W}_n$ for n complex variables. The principle result of this characterization, which involves polynomial expansions of entire functions, is

$$W > \mathscr{W}_2 \geqq \mathscr{W}_3 \geqq \cdots .$$

To simplify notation, the presentation here is given for functions of two variables.

An exact determination of W was obtained by M. A. Evgrafov in 1954 [3]. The determination involves the Gončarov polynomials, defined recursively by

$$G_0(z) = 1 ,$$

$$(1.1)\quad G_n(z; z_0, z_1, \cdots, z_{n-1}) = \frac{z^n}{n!} - \sum_{k=0}^{n-1} \frac{z_k^{n-k}}{(n-k)!} G_k(z; z_0, z_1, \cdots, z_{k-1}) .$$

Let

$$H_n = \max | G_n(0; z_0, \cdots, z_{n-1}) | ,$$

where the maximum is taken over all sequences $\{z_k\}_{k=0}^{n-1}$ whose terms lie on $|z| = 1$. Evgrafov proved that

$$W = \left\{\limsup_{n\to\infty} H_n^{1/n}\right\}^{-1} .$$

An improvement of this result and further characterizations of W were furnished by J. D. Buckholtz [1]. Using properties of the Gončarov polynomials, Buckholtz proved that

$$(1.2)\qquad (.4)^{1/n} H_n^{-1/n} < W \leqq H_n^{-1/n} ,$$

for $n = 1, 2, 3, \cdots$. A consequence of these bounds is