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BLOCKS AND F-CLASS ALGEBRAS OF FINITE GROUPS

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For an arbitrary field F of characteristic $p \geqq 0$, the usual partitioning of the p-regular elements of a finite group G into F-classes (F-conjugacy classes) is extended to all of G in such a way that the F-classes form a basis of a subalgebra Y of the class algebra Z of G over F. The primitive idempotents of $E \otimes_F Y$, where E is an algebraic closure of F, are the same as those of Z. By means of this fact it is shown that if $p > 0$ the number of blocks of G over F with a given defect group D is not greater than the number of p-regular F-classes L of G with defect group D such that the F-class sum of L in Z is not nilpotent; equality holds if $O_{p,p',p}(G) = G$ or if D is Sylow in G. The results are generalized to arbitrary twisted group algebras of G over F.

1. Introduction. The representation theory of a finite group G over an arbitrary field F involves certain subsets of G called F-conjugacy classes or simply F-classes [6, p. 164], [9, p. 306]. In this paper we show (Theorem 4) that the F-class sums in the group algebra A of G over F form a basis of a subalgebra $Y(A)$ of the center $Z(A)$ of A; we may call $Y(A)$ the F-class algebra of G. (If F has prime characteristic p, the definition of the p-singular F-classes requires some care.) The crucial property of $Y(A)$, from our standpoint, is that its extension $Y(A)^E$ to an algebra over an algebraic closure E of F has precisely the same primitive idempotents as the F-algebra $Z(A)$ (Theorem 4); thus the blocks of G over F correspond to the primitive idempotents of an algebra over an algebraically closed field. Furthermore we obtain a corresponding result for any twisted group algebra (without any normalization of the factor set) of G over F by the methods of [16].

We make use of F-class algebras in conjunction with methods of Berman and Bovdi (Bódi) [2], [3] to obtain results about the number of blocks of twisted group algebras. In the group-algebra case these results (Theorems 6, 8, and 9) can be summarized as follows.

THEOREM 1. *Let F have prime characteristic p. For any p-subgroup D of G, the number of blocks of G over F with D as a defect group is less than or equal to the number of p-regular F-classes L of G with D as a defect group such that the F-class sum of L is not a nilpotent element of A. Equality holds here if $O_{p,p',p}(G) = G$*