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# ERGODIC AUTOMORPHISMS AND AFFINE TRANSFORMATIONS OF LOCALLY COMPACT GROUPS

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**It has been conjectured that if a locally compact group *G* has a *continuous automorphism* which is ergodic with respect to Haar measure then *G* must be compact. This is true when *G* is commutative or connected. In this paper further results in support of this conjecture are presented. In particular, it is shown that the problem can be reduced to the consideration of compactly generated, totally disconnected, locally compact groups without compact, open, normal subgroups and that the conjecture holds for many automorphisms of a certain class of such groups. Finally, the structure of locally compact groups which admit ergodic affine transformations is investigated.**

The question of the existence of ergodic automorphisms on non-compact groups was first raised by P. Halmos [5, p. 29]. The commutative case was studied in [10] and [15] and the connected case in [8] and [14]. Theorem 1.1 below has been announced without proof in [16], and some of the results in § 2 have been obtained independently and by somewhat different methods by R. Sato [11], [12] and by N. Aoki and Y. Ito [1].

1. **Automorphisms.** Let $G$ be a locally compact group and $T$ an ergodic (whence bi-continuous and measure preserving, as shown in [10]) automorphism of $G$. Thus, if $\lambda$ denotes a left Haar measure on $G$, $\lambda(A) = 0$ or $\lambda(A^c) = 0$ for any measurable subset $A$ of $G$ such that $T(A) = A$. Let $G_0$ denote the identity component of $G$.

THEOREM 1.1. *If $G/G_0$ is compact then $G$ must be compact. Thus if there exists a noncompact group with an ergodic automorphism then there exists a noncompact totally disconnected one.*

*Proof.* If $G/G_0$ is compact there is a unique maximal normal compact subgroup $N$ of $G$ such that $G/N$ is a Lie group [9, p. 175], [6, § XV. 3]. Since $N$ must be invariant under $T$, there is induced an ergodic automorphism $\tilde{T}$ of $G/N$ (cf. [8]). Since $(G/N)_0$ is open and invariant under $\tilde{T}$, $G/N$ is in fact connected, whence compact. Thus $G$ is compact.

If $G$ is a noncompact group with ergodic automorphism $T$, then by the above the totally disconnected group $G/G_0$ must be noncompact,