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ON COMPLETELY HAUSDORFF-COMPLETION OF A COMPLETELY HAUSDORFF SPACE

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R. M. Stephenson, Jr. (Trans. Amer. Math. Soc. 133 (1968), 537-546) has established the existence of a completely Hausdorff-closed extension X' of an arbitrary completely Hausdorff space X. Stephenson demonstrates that X' enjoys many interesting properties of the Stone-Čech compactification. This paper shows that, by a modification of the topology, X' is made also to possess a property which is in the line of the celebrated property of the Stone-Čech compactification of a completely regular Hausdorff space that it is the largest amongst all Hausdorff compactifications.

1. **Introduction.** A topological space X is called completely Hausdorff if for every pair x, y of distinct points of X there exists a continuous real valued function f on X such that $f(x) \neq f(y)$. A completely Hausdorff space is called completely Hausdorff-closed if every homeomorphic image of it in any completely Hausdorff space is closed. A space Y is termed a completely Hausdorff-closed extension of a completely Hausdorff space X if X is dense in Y and Y is completely Hausdorff-closed. R. M. Stephenson, Jr. in [4] has established the existence of a completely Hausdorff-closed extension (referred to as the completely Hausdorff-completion) X' of an arbitrary completely Hausdorff space X. If X is completely regular (which, of course, assumes Hausdorff property and is necessarily completely Hausdorff) then X' is the Stone-Čech compactification of X. Stephenson shows {[4], Theorem 4} that, even if X is completely Hausdorff but not necessarily completely regular, X' continues to enjoy many interesting properties of the Stone-Čech compactification. By enlarging the topology of X' we shall, in fact, strengthen Theorem 4 of [4] in the sense that property (vii) therein will be replaced by the following:

X' is a projective maximum in the class of completely Hausdorff-closed extensions Y of X with the property that any element in $F(X)$, the set of all continuous functions on X into $[0, 1]$, admits an extension to $F(Y)$.

The above property is, obviously, akin to the well-known fact that the Stone-Čech compactification is largest among the Hausdorff compactifications of a completely regular Hausdorff space.

2. **Notations and definitions.** We shall try to follow the notations and definitions of [4] as far as possible.