PACIFIC JOURNAL OF MATHEMATICS
Vol. 38, No. 1, 1971

# EXAMPLES CONCERNING SUM PROPERTIES FOR METRIC-DEPENDENT DIMENSION FUNCTIONS

Dedicated to Professor J. H. Roberts on the occasion of his sixty-fifth birthday.

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**Let $d_0$ denote the metric dimension function defined by Katětov, and let dim be the covering dimension function. K. Nagami and J. H. Roberts introduced the metric-dependent dimension functions $d_2$ and $d_3$, and J. C. Smith defined the functions $d_6$ and $d_7$. The following relations hold for all metric spaces $(X, \rho)$:**

$$d_2(X, \rho) \leqq d_3(X, \rho) \leqq d_6(X, \rho) \leqq d_7(X, \rho) \leqq d_0(X, \rho) .$$

**Since all of the metric-dependent dimension functions above satisfy a "Weak Sum Theorem," it is natural to ask if any of these functions satisfy the Finite Sum Theorem or the Countable Sum Theorem. In this paper the authors obtain new properties of these dimension functions, and using these results construct examples for which none of the metric dependent dimension functions satisfy either of the sum theorems in question.**

Let $d_0$ denote the metric dimension function defined by Katětov [2], and let dim be the covering dimension function. K. Nagami and J. H. Roberts [5] introduced the metric-dependent dimension functions $d_2$ and $d_3$, and J. C. Smith [7] defined the functions $d_6$ and $d_7$. The following relations hold for all metric spaces $(X, \rho)$:

$$(*) \qquad d_2(X, \rho) \leqq d_3(X, \rho) \leqq d_6(X, \rho) \leqq d_7(X, \rho) \leqq d_0(X, \rho) .$$

In [8] J. C. Smith has shown that all of the above dimension functions satisfy the "Weak Sum Theorem" stated below for $d_2$.

THEOREM. *Let $(X, \rho)$ be a metric space satisfying these conditions:*

(1) $X = \bigcup_{\alpha \in A} F_\alpha$, *where each $F_\alpha$ is closed in $X$.*
(2) $\{F_\alpha : \alpha \in A\}$ *is locally finite.*
(3) $d_2(F_\alpha, \rho) \leqq n$ *for each $\alpha \in A$.*
(4) $\dim [(\mathrm{bdry}\, F_\alpha) \cap F_\beta] \leqq n - 1$ *for $\alpha \neq \beta$.*

*Then $d_2(X, \rho) \leqq n$.*

It is now natural to ask the following question. Do any of the above dimension functions satisfy the Countable Sum Theorem or the Finite Sum Theorem? In this paper we answer this question in the