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A REPRESENTATION FOR THE LOGARITHMIC DERIVATIVE OF A MEROMORPHIC FUNCTION

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A new representation is developed for the logarithmic derivative of a meromorphic function f in terms of its zeros and poles, using as parameters some of the critical points of f. Applications are made to locating all but a finite number of critical points of f.

1. The principal result.

THEOREM 1.1. *Let f be a meromorphic function of finite order ρ possessing the finite zeros $a_1, a_2, a_3, \cdots$ and poles $b_1, b_2, b_3, \cdots$. Let $\zeta_1, \zeta_2, \cdots \zeta_n$, be any $n = [\rho]$ distinct zeros of the derivative f' of f which are not also zeros of f. Then for $z \neq a_j, b_j$ $(j = 1, 2, 3, \cdots)$*

$$\frac{f'(z)}{f(z)} = \sum_{j=1}^{\infty} \frac{\psi(z)}{\psi(a_j)(z - a_j)} - \sum_{j=1}^{\infty} \frac{\psi(z)}{\psi(b_j)(z - b_j)} \tag{1.1}$$

where $\psi(z) = 1$ *for* $n = 0$,

$$\psi(z) = (z - \zeta_1)(z - \zeta_2) \cdots (z - \zeta_n) \qquad \textit{for } n > 0 . \tag{1.2}$$

In (1.1) *the convergence is uniform on every compact set excluding all the a_j and b_j.*

In the case that f is a rational function with m zeros and p poles, identity (1.1) reduces to the familiar formula

$$f'(z)/f(z) = \sum_{j=1}^{m} (z - a_j)^{-1} - \sum_{j=1}^{p} (z - b_j)^{-1} .$$

Furthermore, if the second summation is omitted in (1.1), identity (1.1) reduces to one which we had previously obtained [See 1] for entire functions of finite order.

2. *Proof.* Being a meromorphic function, f can be written as a ratio of two entire functions, each of which has an Hadamard representation in terms of its zeros. Thus,

$$f(z) = z^m e^{P(z)} \prod_{j=1}^{\infty} [E(z/a_j, p)/E(z/b_j, q)] \tag{2.1}$$

where m is an integer (positive, negative or zero); $P(z)$ is a polynomial of degree at most $n = [\rho]$; p and q are nonnegative integers not exceeding n and