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A SUFFICIENT CONDITION FOR L^p-MULTIPLIERS

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Suppose $1 \leqq p \leqq \infty$. For a bounded measurable function ϕ on the n-dimensional euclidean space R^n define a transformation T_ϕ by $(T_\phi f)^\wedge = \phi \hat{f}$, where $f \in L^2 \cap L^p(R^n)$ and $\hat{f}$ is the Fourier transform of f:

$$\hat{f}(\xi) = \hat{f} \frac{1}{\sqrt{2\pi}^n} \int_{R^n} f(x) e^{-i\xi x}\, dx .$$

If T_ϕ is a bounded transform of $L^p(R^n)$ to $L^p(R^n)$, ϕ is said to be L^p-multiplier and the norm of ϕ is defined as the operator norm of T_ϕ.

THEOREM 1. Let $2n/(n+1) < p < 2n/(n-1)$ and ϕ be a radial function on R^n, so that, it does not depend on the arguments and may be denoted by $\phi(r)$, $0 \leqq r < \infty$. If $\phi(r)$ is absolutely continuous and

$$M = \| \phi \|_\infty + \left(\sup_{R>0} R \int_R^{2R} \left| \frac{d}{dr} \phi(r) \right|^2 dr \right)^{1/2} < \infty ,$$

then ϕ is an L^p-multiplier and its norm is dominated by a constant multiple of M.

To prove this theorem we introduce the following notations and Theorem 2. For a complex number $\delta = \sigma + i\tau$, $\sigma > -1$, and a reasonable function f on R^n the Riesz-Bochner mean of order δ is defined by

$$s_R^\delta(f, x) = \frac{1}{\sqrt{2\pi}^n} \int_{|\xi|<R} \left(1 - \frac{|\xi|^2}{R^2}\right)^\delta \hat{f}(\xi) e^{i\xi x} d\xi .$$

Put

$$t_R^\delta(f, x) = s_R^\delta(f, x) - s_R^{\delta-1}(f, x)$$

and define the Littlewood-Paley function by

$$g_\delta^*(f, x) = \left(\int_0^\infty \frac{|t_R^\delta(f, x)|^2}{R} dR \right)^{1/2} ,$$

which is introduced by E. M. Stein in [3]. Then we have the following.

THEOREM 2. *If* $2n/(n + 2\sigma - 1) < p < 2n/(n - 2\sigma + 1)$ *and* $1/2 < \sigma < (n+1)/2$, *then*

$$A \| g_\sigma^*(f) \|_p \leqq \| f \|_p < A' \| g_\sigma^*(f) \|_p ,$$