PACIFIC JOURNAL OF MATHEMATICS
Vol. 38, No. 1, 1971

ON THE NONEQUIVALENCE OF CONSERVATIVE HAUSDORFF METHODS AND HAUSDORFF MOMENT SEQUENCES

J. R. EDWARDS AND S. G. WAYMENT

In this paper we give a counter example to the theorem: A Hausdorff method is convergence preserving if and only if it is generated by a moment sequence as stated in "vector-valued summability methods on a linear normed space" by L. C. Kurtz and D. H. Tucker, Proc. Amer. Math. Soc. 16 (1965) 419–428.

New results are also obtained which extend those known on the equivalence of the generalized Hausdorff moment problem with a generalized Riesz Representation Theorem, and a class of normed spaces is given in which the above mentioned does hold. The key tool in establishing these is the v-integral.

1. Introduction. In [4] Kurtz and Tucker consider summability methods in the setting of linear normed spaces. In that paper they establish an equivalence between a form of the Hausdorff moment problem and an integral representation theorem (Tucker [8]). In this paper we give a stronger formulation of the Hausdorff moment problem and establish its equivalence to the v-integral representation theorem in [1] in the setting of convex spaces. Also in [4] the authors claim to show that a Hausdorff method is regular if and only if it is generated by a moment sequence. However, the proof of sufficiency establishes only that a Hausdorff method generated by a moment sequence is weakly convergence preserving. Goodrich [3] and Remanujan [6] have shown, independenty, in the setting of convex spaces that a Hausdorff method is weakly convergence preserving if and only if it is generated by a weak moment sequence. In §4 of this paper we give an example which shows that in general it is *not* the case that a Hausdorff method generated by a moment sequence is convergence preserving. The remainder of the paper is devoted to obtaining sufficient conditions for a Hausdorff method to be convergence preserving, and we conclude by defining a class of normed spaces in which being generated by a moment sequence is both necessary and sufficient for a Hausdorff method to be convergence preserving.

In the first three sections of this paper, X will denote an F-space and Y a convex space unless otherwise stated. $L[X, Y]$ will denote the collection of continuous linear operators from X into Y, and C