PACIFIC JOURNAL OF MATHEMATICS
Vol. 38, No. 2, 1971

RADON-NIKODÝM THEOREMS FOR THE BOCHNER AND PETTIS INTEGRALS

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The first Radon-Nikodým theorem for the Bochner integral was proven by Dunford and Pettis in 1940. In 1943, Phillips proved an extension of the Dunford and Pettis result. Then in 1968–69, three results appeared. One of these, due to Metivier, bears a direct resemblence to the earlier Phillips theorem. The remaining two were proven by Rieffel and seem to stand independent of the others. This paper is an attempt to put these apparently diverse theorems in some perspective by showing their connections, by simplifying some proofs and by providing some modest extensions of these results. In particular, it will be shown that the Dunford and Pettis theorem together with Rieffel's theorem directly imply Phillips' result. Also, it will be shown that, with almost no sacrifice of economy of effort, the theorems here can be stated in the setting of the Pettis integral.

For ease of reference the theorems mentioned above are listed below in a form convenient for our purposes. Throughout this paper (Ω, Σ, μ) is a finite measure space and X is a Banach space.

I. (*Dunford-Pettis*) [2, *VI.* 8. 10] *Let* $t: L^1(\Omega, \Sigma, \mu) \rightarrow X$ *be a weakly compact operator whose range is separable. The there exists an essentially bounded strongly measurable* $g: \Omega \rightarrow X$ *such that*

$$t(f) = \text{Bochner} - \int_\Omega fg d\mu \quad f \in L^1(\Omega, \Sigma, \mu) .$$

II. (*Phillips*) [6, *p.* 134]. *A vector measure* $F: \rightarrow X$ *is of the form* $F(E) = \text{Bochner} - \int_E f d\mu$, $E \in \Sigma$, *for some Bochner integrable* $F: \Omega \rightarrow X$ *if* F *is* μ*-continuous,* F *is of bounded variation and for each* $\varepsilon > 0$ *there exists* $E_\varepsilon \in \Sigma$ *with* $\mu(\Omega - E_\varepsilon) < \varepsilon$ *such that*

$$\{F(E)/\mu(E): E \subset E_\varepsilon, \mu(E) > 0, E \in \Sigma\}$$

is contained in a weakly compact subset of X.

III. *Metivier* [5]. *The converse of Phillips' theorem is true.*

IV. (*Rieffel*) [7]. *A vector measure* $F: \Sigma \rightarrow X$ *is of the form*