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SUBALGEBRA SYSTEMS OF POWERS OF PARTIAL UNIVERSAL ALGEBRAS

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A set A and an integer $n > 1$ are given. S is any family of subsets of A^n. Necessary and sufficient conditions are found for the existence of a set F of finitary partial operations on A such that S is the set of all subalgebras of $\langle A; F\rangle^n$. As a corollary, a family E of equivalence relations on A is the set of all congruences on $\langle A; F\rangle$ for some F if and only if E is an algebraic closure system on A^2.

For any partial universal algebra, the subalgebras of its nth direct power form an algebraic lattice. The characterization of such lattices for the case $n = 1$ was essentially given by G. Birkhoff and O. Frink [1]. For the case $n = 2$, the characterization was given by the author [4] (see also [3]). The connection between the subalgebra lattices of partial universal algebras and their direct squares was described by the author [5].

In the present paper we are concerned with the subalgebra systems from the following point of view: given a set A and a positive integer n, which systems of subsets of A^n are the subalgebra systems of $\langle A; F\rangle^n$ for some set of partial operations F on A? The problem where F is required to be full is Problem 19 of G. Gratzer [2]. For $n = 1$, such systems are precisely the algebraic closure systems on A[1]. The description of the case $n \geqq 2$ is given here by the Characterization Theorem. We also show that there are partial universal algebras $\langle A; F\rangle$ such that the subalgebra system of $\langle A; F\rangle^2$ is not equal to the subalgebra system of $\langle A; G\rangle^2$ for any set of full operations G. The methods of this paper can be modified to get similar results for infinitary partial algebras, the arities of whose operations are less than a given infinite ordinal.

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1. Let A be a set and n be a positive integer. The set of all functions from $\{1, \cdots, n\}$ into A will be denoted by A^n. If F is a set of finitary partial operations on A, the partial algebra structure obtained on A will be denoted by $\langle A; F\rangle$. By $\langle A; F\rangle^n$ we will mean the partial algebra $\langle A^n; F\rangle$ such that if $f \in F$ is an m-ary partial operation and $a_1, \cdots, a_m \in A^n$ then $a_1 \cdots a_m f$ is defined and equal to $a \in A^n$ if and only if $a_1(j) \cdots a_m(j)f$ is defined and is equal to $a(j)$ for all $1 \leqq j \leqq n$. By a subalgebra of a partial algebra $\langle A; F\rangle$ we will mean