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THE POLYNOMIAL OF A NON-REGULAR DIGRAPH

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This paper studies the matrix equation $f(Z) = D + \lambda J$ where f is a polynomial, Z a square $(0, 1)$-matrix, D is diagonal, $\lambda \neq 0$ and J is the matrix of ones. If Z is thought of as the incidence matrix of a digraph G, the equation implies various path length properties for G. It is shown that such a graph is an amalgamation of regular subgraphs with similar path length properties. Necessary and sufficient parameter conditions on the matrix Z are given in order that it satisfy such an equation for a fixed polynomial f and all non-regular digraphs corresponding to quadratic polynomials f are found.

1. Introduction. The concept of the polynomial of a graph was introduced by Hoffman [3] for regular, connected, non-oriented graphs, and discussed by Hoffman and McAndrew [4] for regular directed graphs. If A is the adjacency matrix of such a graph G, the polynomial of G is taken to be the polynomial $p(x)$ of least degree with $p(A) = J$, the matrix of ones. In extending this notion to non-regular, directed graphs we are concerned with the matrix equation

$$f(Z) = D + \lambda J \tag{1.1}$$

where f is a polynomial, Z a square $(0, 1)$ matrix, D a diagonal matrix and $\lambda \neq 0$. Given (1.1) the conditions: (a) Z has constant row sums; (b) D is a scalar matrix; (c) Z has constant line sums; and (d) The graph of Z is regular; are easily seen to be equivalent. The regular case of (1.1) embraces such studies as the (v, k, λ)-problem [7], (n, k, λ)-systems on k and $k + 1$ [1], Moore graphs [4], strongly regular graphs [9, 10, 11, 12] and even the algebraic studies of central groupoids and universal algebras]2], [6].

In [8] Ryser opens the non-regular question by considering (1.1) with $f(x) = x^2$, and finding all *nonregular* solutions.

The case in which f of (1.1) has degree two is particularly inter-

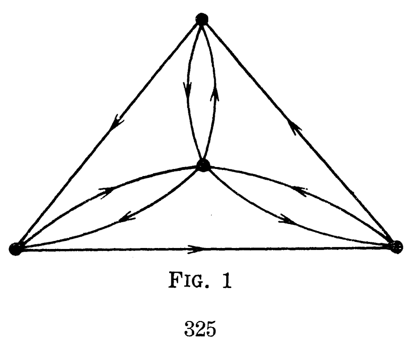

FIG. 1