PACIFIC JOURNAL OF MATHEMATICS
Vol. 38, No. 2, 1971

RESTRICTION OF THE PRINCIPAL SERIES OF $SL(n, C)$ TO SOME REDUCTIVE SUBGROUPS

NGUYEN-HUU-ANH

Let $n = n_1 + \cdots + n_r$, where $r \geqq 2$ and the $n_i's$ are positive integers. Then every element of $G = SL(n, C)$ can be written as a block matrix $(g_{ij})_{1\leqq i,\, j\leqq r}$, where each block g_{ij} is a $n_i \times n_j$ matrix. Let $G_{n_1,\cdots,n_r}$ denote the subgroup of all diagonal block matrices, i.e., g_{ij} is the 0-matrix for $i \neq j$. Let T^χ be any element of the non-degenerate principal series of G. The main purpose of this paper is to decompose the restriction of T^χ to $G_{n_1,\cdots,n_r}$ into irreducible representations.

As we shall see by an induction argument, it is sufficient to consider the restriction of T^χ to $G_{n-1,1}$. Now by the Frobenius reciprocity theorem this restriction problem is equivalent to the decomposition of the induced representations to G of some irreducible representations of $G_{n-1,1}$. Note that

$$G_{n-1,1} \subset G_0 = \{(g_{ij})_{1\leqq i,j\leqq n} \in G \mid g_{in} = 0,\ 1 \leqq i \leqq n-1\}\ ,$$

and hence those induced representations may be obtained by inducing some representations W of G_0. The W's are in turn equivalent to the restrictions of the elements of the non-degenerate principal series to G_0. Therefore they are all irreducible according to Gelfand and Naimark [3], and in fact are divided into n distinct classes of irreducible representations of G_0 [4]. The problem is now completed by applying again the Frobenius reciprocity theorem. It turns out that this restriction problem is equivalent to the problem of decomposing the tensor product of an element of the nondegenerate and an element of the degenerate principal series of G. In fact Theorem 4.2 gives the decompositions of such tensor products in terms of the nondegenerate principal series only. The results contained in this paper were parts of the author's thesis at the University of California, Los Angeles. The author would like to express his gratitude to Professor Donald G. Babbitt for guiding the preparation of the thesis. The author would also like to thank the referee for many helpful suggestions.

1. Some results on induced representations and the Frobenius reciprocity theorem. In this section we shall recall some results on induced representations due to Mackey ([5], [6]) and then prove some