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ON THE MULTIVALENCE OF A CLASS OF MEROMORPHIC FUNCTIONS

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The estimates are given for the radius of the largest disk about the origin on which all the functions of the form $a/z+\phi(z)$ are p-valent, where $\phi(z)$ is an analytic function defined in the unit disk and of modulus less than unity there. Similar results are obtained concerning the starlikeness of the image of circles about the origin.

Suppose a polynomial of degree n assumes p-times a certain value in the center of a circle and does not take this value elsewhere in this circle. The author determines the largest concentric circle in which the polynomial is p-valent. The same problem is then considered in a more general setting and similar results are obtained.

1. Introduction. In this paper we shall deal with two questions. The first part deals with the multivalence of meromorphic functions of the form

$$f(z) = \sum_{k=1}^{n} \frac{A_k}{z - a_k} , \tag{1}$$

where $A_k > 0$ and $a_k \in S$, $k = 1, 2, \cdots, n$; S being the closed unit disk.

Actually most of the results will be deduced for a more general class of functions, namely those representable in the form

$$g(z) = \frac{a}{z} + \phi(z),\ a > 0 , \tag{2}$$

defined for all z such that $0 < |z| < 1$ and such that the function $\phi(z)$ is analytic and of modulus less than unity in the unit disk.

The relation between the functions of the form (1) and (2) becomes clearer if we apply a result due to J. L. Walsh and the author [11] (see loc. cit. Theorem C), according to which the function $f(z)$ defined by (1) can also be written as

$$f(z) = \frac{A}{z - \alpha(z)} , \tag{3}$$

where $A = \sum_{k=1}^{n} A_k$ and where $\alpha(z)$ is an analytic function defined for all z such that $|z| > 1$ and satisfying $|\alpha(z)| < 1$ there. Obviously the functions of the type (2) can now be obtained from (3) by simple transformations.

R. Distler [5] determined the domain of univalence for functions