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OSCILLATORY PROPERTIES OF SOLUTIONS OF EVEN ORDER DIFFERENTIAL EQUATIONS

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Consider the following nth order nonlinear differential equation

(1) $$x^{(n)} + f(t, x, x', \cdots, x^{(n-1)}) = 0 .$$

All functions considered will be assumed continuous and all the solutions of (1), continuously extendable throught the entire nonnegative real axis. A nontrivial solution of (1) is called oscillatory if it has zeros for arbitrarily large t and equation (1) is called oscillatory if all of its solutions are oscillatory. A nontrivial solution of (1) is called nonoscillatory if it has only a finite number of zeros on $[t_0, \infty)$ and equation (1) is called nonoscillatory if all of its solutions are nonoscillatory. In this paper, theorems on oscillation and nonoscillation are presented.

Recently, J. S. W. Wong [8] posed a definition called strongly continuous with which he proved some theorems to (1) for $n = 2$. The proof is based on that of his earlier results [7]. We introduce more general definition. A function $f(t, x_1, \cdots, x_n)$ is called generalized strongly continuous from the left at x_{1c} if $f(t, x_1, x_2, \cdots, x_n)$ is jointly continuous in t and $x_i (i = 1, 2, \cdots, n)$ and for $\varepsilon > 0$ there exist $\delta > 0$, $T \geqq 0$, and $x_\delta \in [x_{1c} - \delta, x_{1c}]$ such that for all $x_1 \in [x_{1c} - \delta, x_{1c}]$, and for all x_i satisfying $|x_i - k_i| \leqq \delta$ (k_i is any constant) for $i = 2, \cdots, T$,

$$(1 - \varepsilon)f(t, x_\delta, k_2, \cdots, k_n) \leqq f(t, x_1, \cdots, x_n) \leqq (1 + \varepsilon)f(t, x_{1c}, k_2, \cdots, k_n) ,$$

for all $t \geqq n$.

Generalized strong continuity from the right is defined analogousely. A function $f(t, x_1, \cdots, x_n)$ is said to be generalized strongly continuous if it is generalized strongly continuous both from the left and from the right. If $f = f(t, x_1)$, then our definition is the same as Wong's one. For example, $f(t, x_1, \cdots, x_n) = a(t)f(x_1, \cdots x_n)$ is generalized strongly continuous.

2. Oscillation and nonoscillation theorems.

THEOREM 1. *Assume that n is even and that*

(α) *$f(t, c, k_2 \cdots, k_n)$ is bounded for any constant c and $k_i (i = 2, \cdots, n)$ and $x_1 f(t, x_1, \cdots, x_n) > 0$ ($x_1 \neq 0$).*

Let $f(t, x_1, \cdots, x_n)$ be generalized strongly continuous from the left