PACIFIC JOURNAL OF MATHEMATICS
Vol. 38, No. 3, 1971

ELEMENTARY SURGERY ALONG A TORUS KNOT

LOUISE MOSER

In this paper a classification of the manifolds obtained by a (p, q) surgery along an (r, s) torus knot is given. If $|\sigma| = |rsp + q| \neq 0$, then the manifold is a Seifert manifold, singularly fibered by simple closed curves over the 2-sphere with singularities of types $\alpha_1 = s$, $\alpha_2 = r$, and $\alpha_3 = |\sigma|$. If $|\sigma| = 1$, then there are only two singular fibers of types $\alpha_1 = s$, $\alpha_2 = r$, and the manifold is a lens space $L(|q|, ps^2)$. If $|\sigma| = 0$, then the manifold is not singularly fibered but is the connected sum of two lens spaces $L(r, s)\#L(s, r)$. It is also shown that the torus knots are the only knots whose complements can be singularly fibered.

1. DEFINITIONS. A *knot* K is a polygonal simple closed curve in S^3 which does not bound a disk in S^3. A *solid torus* T is a 3-manifold homeomorphic to $S^1 \times D^2$. The boundary of T is a *torus*, a 2-manifold homeomorphic to $S \times S^1$. A *meridian* of T is a simple closed curve on ∂T which bounds a disk in T but is not homologous to zero on ∂T. *A meridianal disk* of T is a disk D in T such that $D \cap \partial T = \partial D$ and ∂D is a meridian of T. A *longitude* of T is a simple closed curve on ∂T which is transverse to a meridian of T and is null-homologous in $\overline{S^3\text{-}T}$. A *meridianlongitude pair for* T is an ordered pair (M, L) of curves such that M is a meridian of T and L is a longitude of T transverse to M. $\pi_1(\partial T) \cong Z \times Z$ with generators M and L. $qM + pL$ is the homotopy class of a simple closed curve on ∂T if and only if p and q are relatively prime.

A *torus knot of type* (r, s), denoted $K(r, s)$, is defined as follows. Let T be a standardly embedded solid torus in S^3, that is, T is isotopic to a regular neighborhood of a polygonal curve in the x-y plane. Then $\overline{S^3\text{-}T}$ is a solid torus. Let J_1 and J_2 be oriented simple closed curves on ∂T such that J_1 bounds a disk in T and J_2 bounds a disk in $\overline{S^3\text{-}T}$, that is J_1 is meridianal and J_2 is longitudinal. Identifying J_1 with $(1, 0)$ and J_2 with $(0, 1)$, let r and s be relatively prime integers, $r > s > 0$, and let $K(r, s)$ be a simple closed curve in (r, s). Then $K(r, s)$ is a torus knot of type (r, s). By Van Kampen's theorem $\pi_1(S^3\text{-}K(r,s)) \cong (a, b \mid a^r = b^s)$.

A space is a *lens space* if it contains a solid torus such that the closure of its complement is also a solid torus. Hence one way to view a lens space is as the space obtained by identifying two solid tori by a homeomorphism on the boundary.

Basic Construction: *Elementary surgery along a knot*. Let N