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CANONICAL DOMAINS AND THEIR GEOMETRY IN C^n

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In this paper we introduce some differential geometric properties of canonical domains of bounded domains in C^n, using our synthetic expression by matrix. In the proofs of the theorems, our formulas of matrix derivatives play the leading part.

In order to construct relatively invariant matrices, the author devised formulas of matrix derivatives and obtained some results ([2]). Here we use these formulas for the calculations on the argument of the theorems of geometry. The constructed matrix $\frac{1}{2}{}_2T_D(\bar{z}, z)$ (see [2]) becomes the *curvature tensor*, and $T_D^{-1}(\bar{z}, z)(\partial T_D(\bar{z}, z)/\partial z)$ becomes *Christoffel symbols* in the Kaehler manifold with the metric $ds_D = dz^* T_D(\bar{z}, z)dz$ where ${}_2T_D(\bar{z}, z) = (E_n \times T_D(\bar{z}, z))(\partial/\partial z^*)(T_D^{-1}(\bar{z}, z)(\partial T_D(\bar{z}, z)/\partial z))$ and $T_D(\bar{z}, z) = \partial^2 log K_D(\bar{z}, z)/\partial z^* \partial z$. We study some differential geometric properties of canonical domains, that is, Bergman representative domains, m-representative domains, homogeneous domains, and our minimal domains of moment of inertia which are defined and investigated in § 2 ([1], [5], [7], [12]).

We calculate Christoffel symbols at the center of canonical domains and give the condition which a geodesic curve through the center of a representative domain satisfies in Theorems 3.4. In Theorems 3.7–12 and Corollaries 3.1–4, we discuss *scalar curvature* and *holomorphic sectional curvature*.

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1. Preliminaries. Let D be a domain in C^n which posses a Bergman kernel function $K_D(\bar{t}, z) \equiv \varphi^*(t)\varphi(z)$, $t, z \in D$, where $\varphi(z) \equiv (\varphi_1(z), \varphi_2(z), \cdots)'$ and the marks $'$ and $*$ denote the transposed and transposed conjugate matrices respectively. We consider a vector function $w(z) \equiv (w_1(z), \cdots, w_n(z))'$ in D. If the function $w(z)$ is both holomorphic and locally one-to-one, i.e., $det(dw/dz) \neq 0$, then the function defines a pseudo-conformal mapping of D onto another domain $\Delta \subset C^n$. Further, the inner product of two functions f, g belonging to a class $\mathscr{L}_D^2$ of all holomorphic functions $\zeta(z)$ of D which satisfy $\int_D |\zeta(z)|^2 dv_D \equiv Sp(\int_D \zeta(z)\zeta^*(z)dv_D) < \infty$, as follows:

$$(f, g)_D \equiv \int_D f(z)g^*(z)dv_D \ ,$$

where dv_D denotes the Euclidean volume element on D. Moreover we